

Calculus · supported focus

Supported focus · HL

## Supported scope

- Supported HL focus. First assessment 2021; current through 2028.  
First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.
- State model conditions and distinguish fictional data from measurements.

# Differentiation · 1.1

## Reasoning

A curved road has different slopes at different positions. An average gradient cannot describe every point. This lesson studies derivative : The instantaneous rate of change, also the gradient of a tangent.

# Differentiation · 2.1

## Reasoning

For  $y = ax^n$ ,  $dy/dx = anx^{n-1}$ . A stationary point satisfies  $dy/dx = 0$ . Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

# Differentiation · 3.1

## Reasoning

Use the displayed relationship within its stated domain.

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

## Differentiation · 3.2

### Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

## Differentiation · 3.3

### Reasoning

For  $y=x^3-3x$ ,  $dy/dx=3x^2-3$ . At  $x=1$ , the gradient is 0 and  $y=-2$ . The second derivative is  $6x$ , positive at  $x=1$ , so this is a local minimum. At  $x=-1$ ,  $y=2$  and the second derivative is negative, giving a local maximum.

# Differentiation · 4.1

## Reasoning

A zero derivative does not always mean a maximum or minimum:  $y=x^3$  is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

## Differentiation · 4.2

### Reasoning

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

# Differentiation · 5.1

## Reasoning

At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

# Differentiation · 6.1

## Reasoning

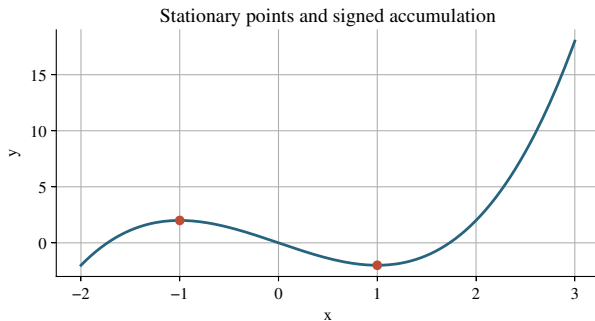
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# Differentiation · 6.2

## Reasoning

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

## Differentiation · diagram



- Original teaching diagram: Derivatives and stationary points
- Use the labels and the stated model together.

## Differentiation · original transfer

## Your turn

For  $y=x^3-3x$  on  $[-2, 2]$ , find the absolute maximum and minimum, including all candidates.

# Differentiation · transfer reasoning

## Method and interpretation

Derivative  $3x^2 - 3 = 0$  at  $\pm 1$ . Values  $y(-2) = -2$ ,  $y(-1) = 2$ ,  $y(1) = -2$ ,  $y(2) = 2$ .

Maximum 2 at  $-1, 2$ ; minimum  $-2$  at  $-2, 1$ . Checking only stationary points misses endpoint ties.

# Integration · 1.1

## Reasoning

A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time? This lesson studies antiderivative : A function whose derivative equals the given integrand.

# Integration · 2.1

## Reasoning

For  $n \neq -1$ , the integral of  $ax^n$  is  $ax^{n+1}/(n+1) + C$ . A definite integral is signed accumulation. Split at sign changes when total area or distance is required. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

## Integration · 3.1

## Reasoning

Use the displayed relationship within its stated domain.

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

# Integration · 3.2

## Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

# Integration · 3.3

## Reasoning

For  $v(t)=3t^2-3$  over  $0 \leq t \leq 2$ , displacement  $= [t^3-3t]_0^2 = 2$ . Since  $v$  changes sign at  $t=1$ , distance  $= -[t^3-3t]_0^1 + [t^3-3t]_1^2 = 2+4=6$ . The constants cancel only for a definite integral.

# Integration · 4.1

## Reasoning

The integral of  $1/x$  is  $\ln|x|+C$ , not the power rule with  $n=-1$ . Signed area can be zero even when the total enclosed area is positive. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

## Integration · 4.2

### Reasoning

A definite integral always equals the total positive area, even when the graph crosses the axis. This claim is false. Explain which definition or assumption it violates.

# Integration · 5.1

## Reasoning

Identify what the integral means and include the correct units. A rate measured per second integrates to the underlying quantity, not to another rate. Differentiate an antiderivative to check it. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

# Integration · 6.1

## Reasoning

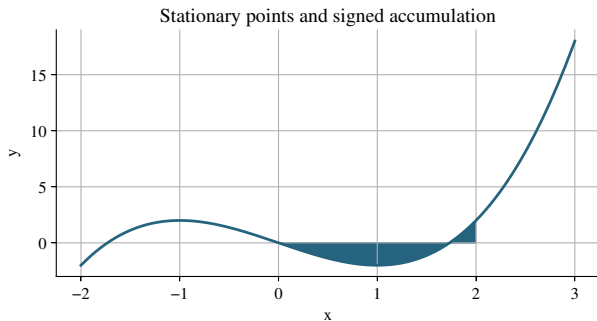
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# Integration · 6.2

## Reasoning

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

# Integration · diagram



- Original teaching diagram: Integrals, area and accumulation
- Use the labels and the stated model together.

## Integration · original transfer

## Your turn

For velocity  $v=t-2$  on  $0 \leq t \leq 5$ , find displacement and total distance and explain the distinction.

# Integration · transfer reasoning

## Method and interpretation

$\text{Integral}[t^2/2 - 2t]_0^5 = 2.5$ . Sign changes at 2; signed integrals are  $-2$  then  $4.5$ , so distance  $6.5$ . Units follow velocity/time units. Distance integrates  $|v|$ .

# Calculus Rules · 1.1

## Reasoning

A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input. This lesson studies chain rule : The rule that multiplies the outer derivative by the inner derivative for a composite function.

# Calculus Rules · 2.1

## Reasoning

For  $y=f(g(x))$ ,  $y' = f'(g(x))g'(x)$ . For  $uv$ , differentiate to  $u'v + uv'$ . For  $u/v$ , use  $(u'v - uv')/v^2$ , where  $v \neq 0$ . State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

# Calculus Rules · 3.1

## Reasoning

Use the displayed relationship within its stated domain.

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

## Calculus Rules · 3.2

### Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

## Calculus Rules · 3.3

## Reasoning

For  $y=(3x+1)^4$ ,  $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$ . At  $x=0$ , the gradient is 12.

For  $x^2+y^2=25$ , differentiate implicitly:  $2x+2y y' = 0$ , so  $y' = -x/y$  when  $y \neq 0$ .

# Calculus Rules · 4.1

## Reasoning

A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of  $y$  brings a  $dy/dx$  factor. A quotient's denominator is squared. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

## Calculus Rules · 4.2

### Reasoning

The derivative of  $u(x)v(x)$  is always  $u$  prime times  $v$  prime. This claim is false. Explain which definition or assumption it violates.

# Calculus Rules · 5.1

## Reasoning

Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

# Calculus Rules · 6.1

## Reasoning

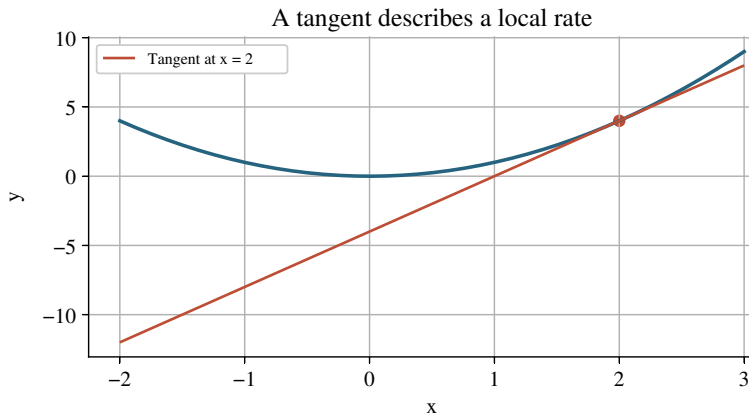
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## Calculus Rules · 6.2

### Reasoning

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

## Calculus Rules · diagram



The tangent slope is the local derivative.

## Calculus Rules · original transfer

### Your turn

For  $x^2 + y^2 = 25$ , find  $dy/dx$  and discuss its values at  $(3, 4)$  and  $(5, 0)$ , using geometry to check.

## Calculus Rules · transfer reasoning

### Method and interpretation

$y' = -x/y$  where  $y \neq 0$ . At  $(3, 4)$ ,  $-3/4$ . At  $(5, 0)$  the tangent is vertical, so finite  $dy/dx$  is undefined; implicit differentiation does not license division by zero.

# Integration Methods · 1.1

## Reasoning

A simple-looking product such as  $xe^x$  cannot be integrated by separately integrating each factor. This lesson studies integration by parts : An integration method based on the derivative of a product.

## Integration Methods · 2.1

### Reasoning

Use substitution when an inner derivative appears as a factor. By parts,  $\int u v' = uv - \int u' v$ . For a rational function, divide first if needed, then decompose into partial fractions. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

## Integration Methods · 3.1

## Reasoning

Use the displayed relationship within its stated domain.

$$\int uv' dx = uv - \int u'v dx$$

## Integration Methods · 3.2

### Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

## Integration Methods · 3.3

### Reasoning

For integral  $x e^x dx$ , take  $u=x$  and  $v \text{ prime}=e^x$ . Then the integral is  $x e^x - e^x + C$ .

For integral  $2x/(x^2+1) dx$ , substitute  $w=x^2+1$ ,  $dw=2x dx$ ; the answer is  $\ln(x^2+1)+C$ .

## Integration Methods · 4.1

### Reasoning

Choosing  $u$  and  $v$  prime well matters: the remaining integral should become simpler. In a definite substitution, either change the limits or return to  $x$  before applying the original limits. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

## Integration Methods · 4.2

### Reasoning

The integral of a product equals the product of its separate integrals. This claim is false. Explain which definition or assumption it violates.

## Integration Methods · 5.1

### Reasoning

Check by differentiating. For volumes of revolution around the  $x$ -axis use  $V = \pi \int y^2 dx$ ; do not confuse the square of a function with the integral of the function. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

## Integration Methods · 6.1

### Reasoning

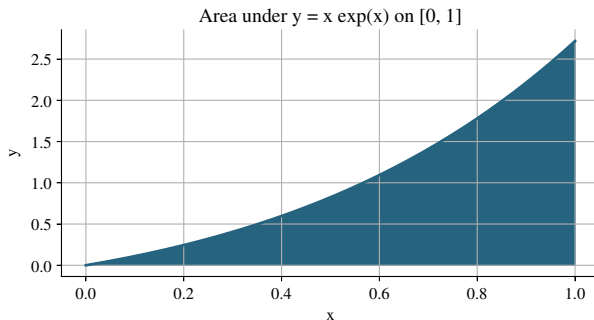
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## Integration Methods · 6.2

### Reasoning

An integration method based on the derivative of a product. Choose the relationship, show the method, check its assumptions and interpret the result.

## Integration Methods · diagram



- Original teaching diagram: Substitution, parts and partial fractions
- Use the labels and the stated model together.

## Integration Methods · original transfer

### Your turn

Evaluate integral of  $x \ln x$  from 1 to  $e$  by parts and differentiate the antiderivative as a check.

# Integration Methods · transfer reasoning

## Method and interpretation

Take  $u=\ln x$ ,  $dv=x$  dx: antiderivative  $(x^2/2)\ln x - x^2/4$ . Bounds give  $(e^2+1)/4$ .  
Differentiation returns  $x\ln x$ ; domain  $x>0$  matters.

# Differential · 1.1

## Reasoning

A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied. This lesson studies initial condition : A specified value that selects a particular solution from a family.

# Differential · 2.1

## Reasoning

For  $dy/dx=ky$ , separate variables:  $dy/y=k dx$ , giving  $y=Ae^{(kx)}$ . Use an initial condition to find  $A$ . Euler's method takes  $y_{next}=y+h f(x,y)$ , with a chosen step  $h$ . State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

## Differential · 3.1

## Reasoning

Use the displayed relationship within its stated domain.

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

## Differential · 3.2

### Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

## Differential · 3.3

### Reasoning

With  $dy/dx=0.5y$  and  $y(0)=2$ ,  $y=2e^{(0.5x)}$ . Euler with  $h=0.2$  gives  $y(0.2)\approx 2+0.2\times 1=2.2$  and  $y(0.4)\approx 2.2+0.2\times 1.1=2.42$ . The exact second value is about 2.4428.

# Differential · 4.1

## Reasoning

An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by  $y$  can omit an equilibrium solution  $y=0$ . Euler accuracy depends on step size and the equation. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

## Differential · 4.2

### Reasoning

Every step size gives exactly the same Euler approximation. This claim is false. Explain which definition or assumption it violates.

# Differential · 5.1

## Reasoning

For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions. This is Further Pure scope; check the named unit. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

# Differential · 6.1

## Reasoning

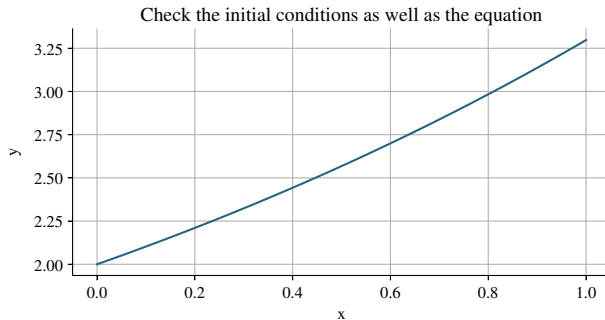
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## Differential · 6.2

### Reasoning

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.

# Differential · diagram



- Original teaching diagram: Differential equations and numerical solutions
- Use the labels and the stated model together.

## Differential · original transfer

### Your turn

Use Euler  $h=0.5$  for  $y'=-y$ ,  $y(0)=2$  to estimate  $y(1)$ . Compare with  $2e^{-1}$  and state a limitation.

# Differential · transfer reasoning

## Method and interpretation

Updates  $2 \rightarrow 1 \rightarrow 0.5$ ; exact  $\approx 0.735759$ . Euler underestimates here; finite steps introduce error. Halve the step and compare convergence, rather than treating the numerical value as exact.

## Check yourself

- Explain derivative using the stated model.
- Explain antiderivative using the stated model.
- Change one value in the original transfer; explain which assumption still matters.