

Number and algebra · supported focus

Supported focus · HL

Supported scope

- Supported HL focus. First assessment 2021; current through 2028.
First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.
- State model conditions and distinguish fictional data from measurements.

Number · 1.1

Reasoning

A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers? This lesson studies prime factor : A prime number that divides the integer exactly.

Number · 2.1

Reasoning

Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Number · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

Number · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Number · 3.3

Reasoning

$72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. Their HCF is $2 \times 9 = 18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5 = 360$.

Number · 4.1

Reasoning

The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Number · 4.2

Reasoning

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

Number · 5.1

Reasoning

For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Number · 6.1

Reasoning

Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective. Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Number · 6.2

Reasoning

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

Number · diagram

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

- Original teaching diagram: Exact arithmetic and estimation
- Use the labels and the stated model together.

Number · original transfer

Your turn

A supplier has 84 pencils and 126 pens. Find the greatest number of identical bags without leftovers and the contents of each; then explain why the LCM answers a different question.

Number · transfer reasoning

Method and interpretation

HCF=42, each bag 2 pencils and 3 pens. LCM=252 is a common multiple, useful for matching repeated quantities, not the greatest number of bags dividing both stocks.

Indices · 1.1

Reasoning

A microscope records a length of 0.000072 metres. A compact representation must keep its size correct. This lesson studies index : The power to which a base is raised.

Indices · 2.1

Reasoning

For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \leq a < 10$. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Indices · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

Indices · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Indices · 3.3

Reasoning

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{(3/4)} = (16^{(1/4)})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

Indices · 4.1

Reasoning

Index laws do not turn a sum into a single power: $2^3+2^4=24$, not 2^7 . Do not round a surd when an exact answer is requested. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Indices · 4.2

Reasoning

For every positive a , $a^2 + a^3$ equals a^5 . This claim is false. Explain which definition or assumption it violates.

Indices · 5.1

Reasoning

Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Indices · 6.1

Reasoning

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Indices · 6.2

Reasoning

The power to which a base is raised. Choose the relationship, show the method, check its assumptions and interpret the result.

Indices · diagram

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

- Original teaching diagram: Indices, surds and standard form
- Use the labels and the stated model together.

Indices · original transfer

Your turn

Solve $x^{3/2}=27$ over the real domain used for this power. Check the result and explain why a negative candidate is invalid.

Indices · transfer reasoning

Method and interpretation

Take the nonnegative real square-root convention: $x \geq 0$. $x = 27^{(2/3)} = 9$; $9^{(3/2)} = 27$.
A negative x has no real square root under this convention.

Sequences · 1.1

Reasoning

A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models. This lesson studies common ratio : The constant multiplier between consecutive terms of a geometric sequence.

Sequences · 2.1

Reasoning

For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{(n-1)}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Sequences · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2} [2a + (n - 1)d]$$

Sequences · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Sequences · 3.3

Reasoning

For $a=5, d=3, n=8$, $u_8=5+7\times 3=26$ and $S_8=8(10+21)/2=124$. For $a=12, r=1/2$, $S_{\text{infinity}}=12/(1-1/2)=24$. For $u_{(n+1)}=2u_n+1$ with $u_1=1$, the next terms are 3, 7, 15.

Sequences · 4.1

Reasoning

The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Sequences · 4.2

Reasoning

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

Sequences · 5.1

Reasoning

Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Sequences · 6.1

Reasoning

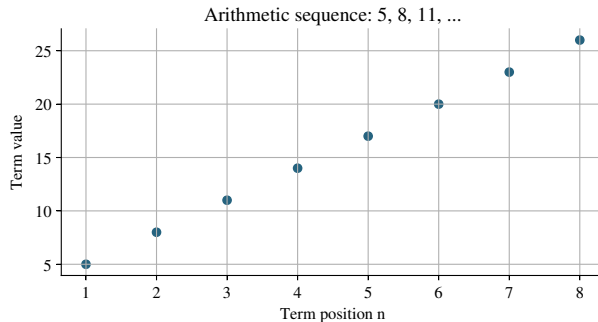
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Sequences · 6.2

Reasoning

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

Sequences · diagram



- Original teaching diagram: Sequences, series and recurrence
- Use the labels and the stated model together.

Sequences · original transfer

Your turn

A sequence obeys $u_1=4$ and $u_{n+1}=0.5 u_n+3$. Find its fixed point and express u_n-6 as a geometric sequence. Does the sum of u_n converge?

Sequences · transfer reasoning

Method and interpretation

Fixed point 6. Subtracting 6 gives $u_{n+1}-6=0.5(u_n-6)$, so $u_n=6-2(0.5)^{n-1}$.

Terms approach 6, so their infinite sum diverges; a convergent sequence need not give a convergent series.

Percent · 1.1

Reasoning

A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price. This lesson studies multiplier : A factor that performs a percentage change in one multiplication.

Percent · 2.1

Reasoning

A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Percent · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

Percent · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Percent · 3.3

Reasoning

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240 / 0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.

Percent · 4.1

Reasoning

A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Percent · 4.2

Reasoning

A 20% decrease followed by a 20% increase restores the starting price. This claim is false. Explain which definition or assumption it violates.

Percent · 5.1

Reasoning

For direct proportion use $y=kx$; for inverse proportion use $y=k/x$. Calculate k from a known pair before using a new value. State what you held constant. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Percent · 6.1

Reasoning

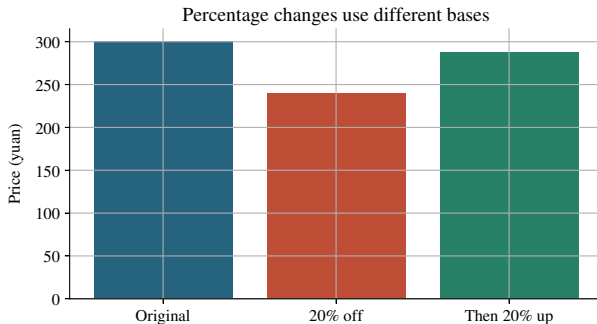
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Percent · 6.2

Reasoning

A factor that performs a percentage change in one multiplication. Choose the relationship, show the method, check its assumptions and interpret the result.

Percent · diagram



- Original teaching diagram: Percentages, ratio and proportional reasoning
- Use the labels and the stated model together.

Percent · original transfer

Your turn

A price rises 20% then falls 20%. Find the net change and the percentage increase needed to recover the original after that pair.

Percent · transfer reasoning

Method and interpretation

Factor $1.2 \times 0.8 = 0.96$, a 4% fall. Recovery factor $1/0.96$, giving 4.1667% increase. Both percentage changes use different bases.

Complex · 1.1

Reasoning

The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots. This lesson studies complex conjugate : The number obtained by reversing the sign of the imaginary part.

Complex · 2.1

Reasoning

Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate $=a^2+b^2$. The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Complex · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

Complex · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Complex · 3.3

Reasoning

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.

Complex · 4.1

Reasoning

The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Complex · 4.2

Reasoning

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

Complex · 5.1

Reasoning

Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Complex · 6.1

Reasoning

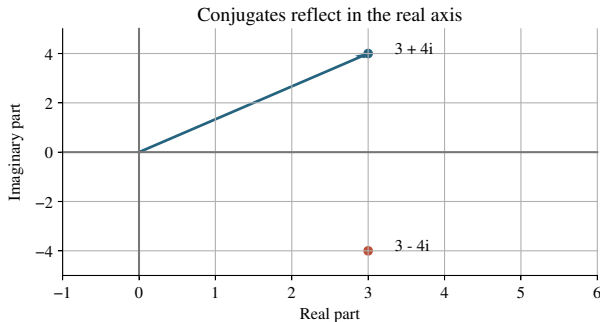
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Complex · 6.2

Reasoning

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

Complex · diagram



- Original teaching diagram: Complex numbers and roots
- Use the labels and the stated model together.

Complex · original transfer

Your turn

Find all complex roots of $z^2 = -8i$ in Cartesian form and verify them.

Complex · transfer reasoning

Method and interpretation

Polar modulus 8 and argument $-\pi/2$ give roots modulus $\sqrt{8}$, arguments $-\pi/4$ and $3\pi/4$: $2-2i$ and $-2+2i$. Squaring each gives $-8i$. Both roots are needed.

Binomial · 1.1

Reasoning

A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects. This lesson studies binomial coefficient :
The number of ways to choose a specified number of objects from a set.

Binomial · 2.1

Reasoning

For positive integer n , $(a+b)^n$ is a finite binomial expansion. For noninteger n , expand $(1+x)^n$ as $1+nx+n(n-1)x^2/2+\dots$ with $|x|<1$. Factor out constants before using this form. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Binomial · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

Binomial · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Binomial · 3.3

Reasoning

$(1+2x)^5 = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$. For $(1+x)^{-1}$, the first three terms are $1 - x + x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1 \approx 0.909091$.

Binomial · 4.1

Reasoning

An expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$.

Positive-integer expansions are finite and do not have that convergence restriction.

When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Binomial · 4.2

Reasoning

Every binomial expansion is a finite polynomial, including powers that are not positive integers. This claim is false. Explain which definition or assumption it violates.

Binomial · 5.1

Reasoning

State the range of validity alongside an approximation. Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x . A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Binomial · 6.1

Reasoning

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Binomial · 6.2

Reasoning

The number of ways to choose a specified number of objects from a set. Choose the relationship, show the method, check its assumptions and interpret the result.

Binomial · diagram

$$(1 + 2x)^5$$

Coefficients: 1, 10, 40, 80, 80, 32

The x^2 coefficient includes $(2x)^2$

- Original teaching diagram: Binomial expansion and valid approximations
- Use the labels and the stated model together.

Binomial · original transfer

Your turn

Use $1-x+x^2$ to approximate $1/(1+x)$ at $x=0.2$. Find the exact error algebraically and explain its sign.

Binomial · transfer reasoning

Method and interpretation

Approximation 0.84; exact $5/6$. Identity $(1-x+x^2)-1/(1+x)=x^3/(1+x)$, so error $1/150 \approx 0.006667$, positive. The infinite geometric expansion requires $|x| < 1$.

Series Advanced · 1.1

Reasoning

Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale. This lesson studies Taylor polynomial : A polynomial formed from a function's derivatives at a chosen centre.

Series Advanced · 2.1

Reasoning

About $x=a$, $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots$ when the expansion is valid. At $a=0$ it is a Maclaurin expansion. Finite sums may also simplify by cancellation or standard sum formulae. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Series Advanced · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

Series Advanced · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Series Advanced · 3.3

Reasoning

The first three terms of e^x are $1+x+x^2/2$. At $x=0.1$ this gives 1.105, close to $e^{0.1} \approx 1.105170$. For the sum of k^2 from $k=1$ to 5, $n(n+1)(2n+1)/6$ gives $5 \times 6 \times 11 / 6 = 55$.

Series Advanced · 4.1

Reasoning

The factorial belongs in the denominator of every Taylor coefficient. The expansion centre is not always zero. An approximation close to its centre can be poor far away. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Series Advanced · 4.2

Reasoning

A Taylor polynomial is always exact for the original function at every input. This claim is false. Explain which definition or assumption it violates.

Series Advanced · 5.1

Reasoning

For a telescoping series, write several terms and identify both surviving ends. For a Taylor approximation, state its order and compare with a known value or a remainder estimate where required. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Series Advanced · 6.1

Reasoning

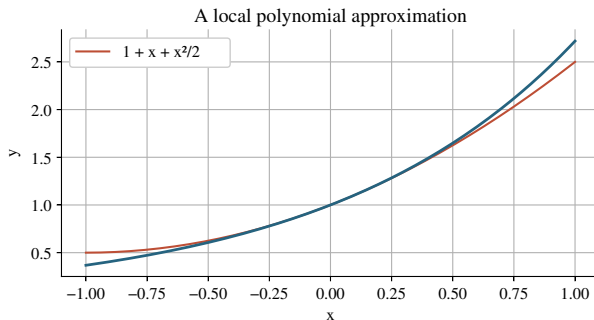
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Series Advanced · 6.2

Reasoning

A polynomial formed from a function's derivatives at a chosen centre. Choose the relationship, show the method, check its assumptions and interpret the result.

Series Advanced · diagram



- Original teaching diagram: Series, finite differences and Taylor expansions
- Use the labels and the stated model together.

Series Advanced · original transfer

Your turn

Approximate $e^{0.2}$ with $1+x+x^2/2$. Give a remainder bound using $e^\xi x^3/6$ with $0 < \xi < 0.2$ and explain what the bound certifies.

Series Advanced · transfer reasoning

Method and interpretation

Approximation 1.22; $0 < R < e^{0.2}(0.2)^3/6 \approx 0.001629$. The quadratic underestimates and the exact value lies between 1.22 and 1.221629. The bound depends on the stated Taylor theorem conditions.

Check yourself

- Explain prime factor using the stated model.
- Explain index using the stated model.
- Change one value in the original transfer; explain which assumption still matters.