

Functions · supported focus

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · HL

Scope and prerequisites

Supported HL focus. First assessment 2021; current through 2028. First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.

Prerequisites: read the stated quantities and units, use arithmetic and the model conditions below. Each lesson develops its own method before independent transfer.

These are original or explicitly fictional teaching examples, not actual measurements or completed assessed learner investigations.

Domains, inverses and composition

■ Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

■ Choose the mathematical structure

- State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

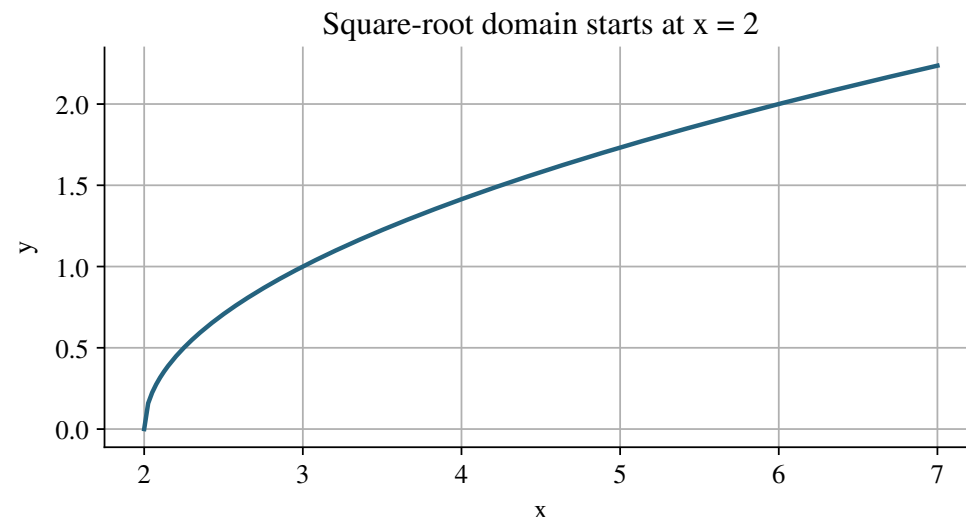
■ Work through a checked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $f(x)=\sqrt{(x-2)}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{(x-2)}$, $x=y^2+2$. Thus f inverse(x)= x^2+2 with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



■ Test a tempting shortcut

- Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every quadratic function on all real numbers has an inverse function. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Check $f(f^{-1}(x))=x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The set of allowed inputs to a function. Choose the relationship, show the method, check its assumptions and interpret the result.

Exponentials, logarithms and modelling

■ Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.

■ Choose the mathematical structure

- For $y=Ae^{(kt)}$, k is a proportional rate. Taking logs gives $\ln y=\ln A+kt$. Logarithms require positive arguments, and $\log(x+y)$ is not $\log x+\log y$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

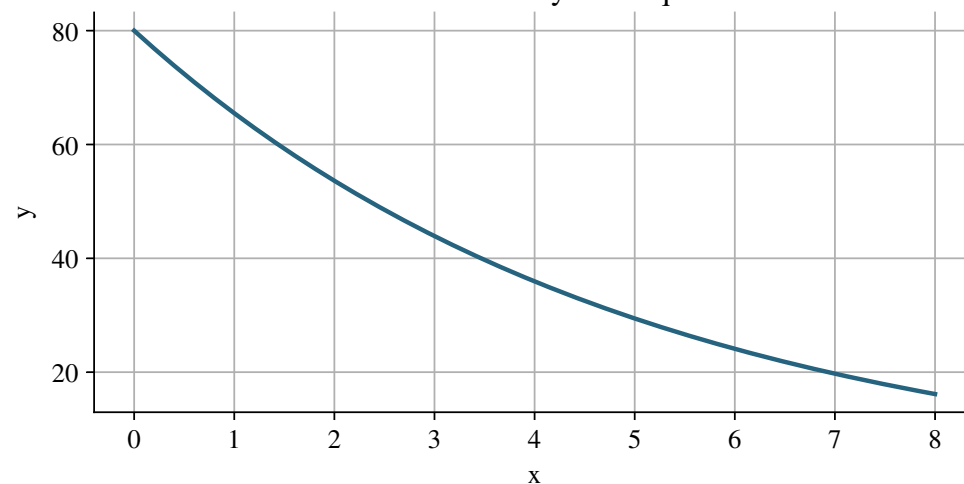
$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

If $y=80e^{(-0.2t)}$, $y=40$ gives $e^{(-0.2t)}=1/2$. Hence $t=\ln 2/0.2\approx 3.466$. For $3^x=20$, $x=\ln 20/\ln 3$. Plotting $\ln y$ against t linearises this exponential model.

Same fractional decay over equal times



■ Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For positive x and y , $\ln(x+y)=\ln x+\ln y$. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.