

Number and algebra · supported focus

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · HL

Scope and prerequisites

Supported HL focus. First assessment 2021; current through 2028. First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.

Prerequisites: read the stated quantities and units, use arithmetic and the model conditions below. Each lesson develops its own method before independent transfer.

These are original or explicitly fictional teaching examples, not actual measurements or completed assessed learner investigations.

Exact arithmetic and estimation

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

■ Choose the mathematical structure

- Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

■ Test a tempting shortcut

- The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

Indices, surds and standard form

■ How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

■ Choose the mathematical structure

- For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \leq a < 10$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{(3/4)} = (16^{(1/4)})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$0.000072 = 7.2 \times 10^{-5}$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

■ Test a tempting shortcut

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For every positive a, $a^2 + a^3$ equals a^5 . This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

The power to which a base is raised. Choose the relationship, show the method, check its assumptions and interpret the result.

Sequences, series and recurrence

■ Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** : The constant multiplier between consecutive terms of a geometric sequence.

■ Choose the mathematical structure

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

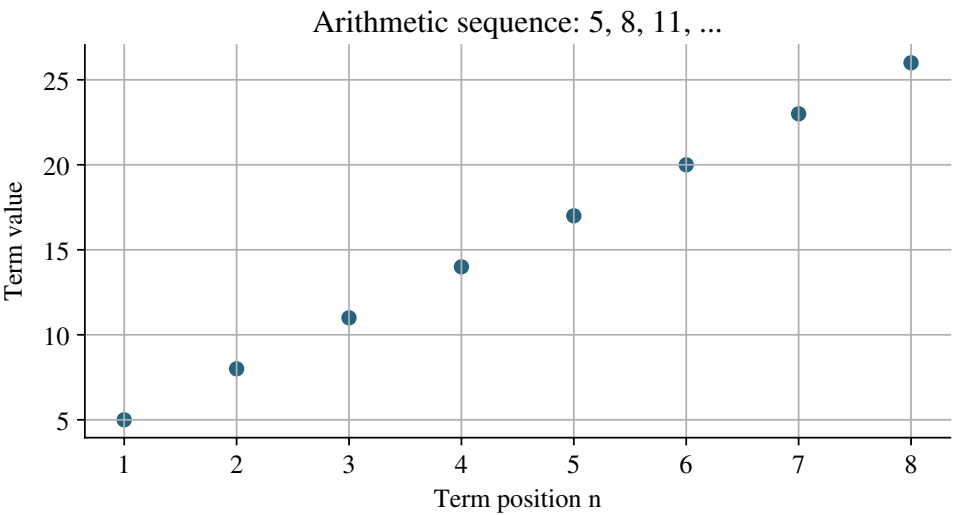
■ Work through a checked case

$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2} [2a + (n - 1)d]$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10 + 21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



■ Test a tempting shortcut

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

Percentages, ratio and proportional reasoning

■ Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier**: A factor that performs a percentage change in one multiplication.

■ Choose the mathematical structure

- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

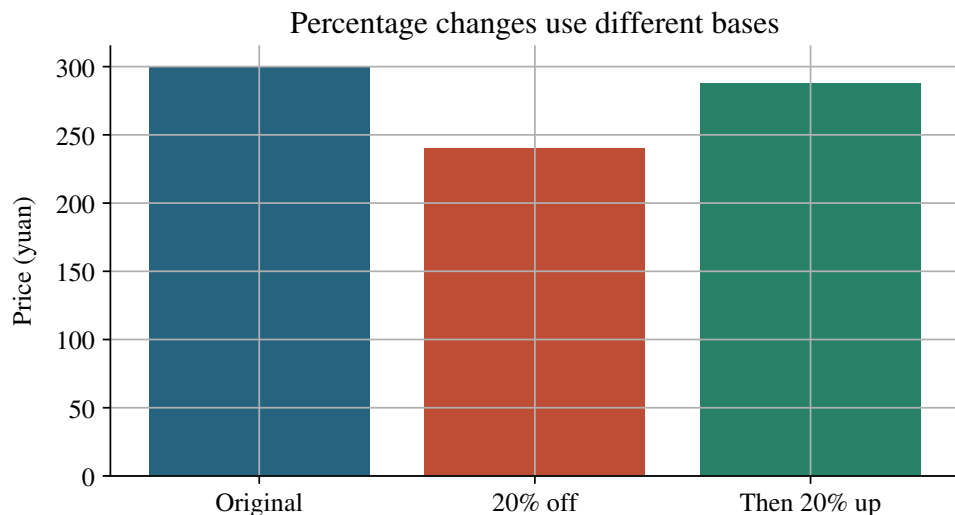
■ Work through a checked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240/0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



■ Test a tempting shortcut

- A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A 20% decrease followed by a 20% increase restores the starting price. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$. Calculate k from a known pair before using a new value. State what you held constant.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

A factor that performs a percentage change in one multiplication. Choose the relationship, show the method, check its assumptions and interpret the result.

Complex numbers and roots

■ What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.

■ Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate $=a^2+b^2$. The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

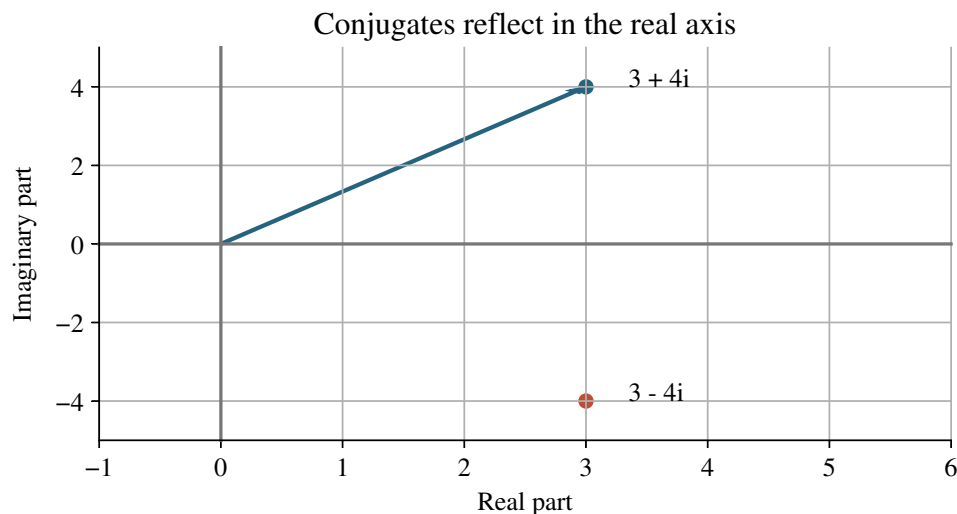
■ Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



■ Test a tempting shortcut

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

Binomial expansion and valid approximations

■ How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** : The number of ways to choose a specified number of objects from a set.

■ Choose the mathematical structure

- For positive integer n , $(a+b)^n$ is a finite binomial expansion. For noninteger n , expand $(1+x)^n$ as $1+nx+\frac{n(n-1)}{2}x^2+\dots$ with $|x|<1$. Factor out constants before using this form.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$(1+2x)^5 = 1+10x+40x^2+80x^3+80x^4+32x^5$. For $(1+x)^{-1}$, the first three terms are $1-x+x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1 \approx 0.909091$.

$$(1 + 2x)^5$$

Coefficients: 1, 10, 40, 80, 80, 32

The x^2 coefficient includes $(2x)^2$

■ Test a tempting shortcut

- An expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$. Positive-integer expansions are finite and do not have that convergence restriction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every binomial expansion is a finite polynomial, including powers that are not positive integers. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- State the range of validity alongside an approximation. Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

The number of ways to choose a specified number of objects from a set. Choose the relationship, show the method, check its assumptions and interpret the result.

Series, finite differences and Taylor expansions

■ Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **Taylor polynomial** : A polynomial formed from a function's derivatives at a chosen centre.

■ Choose the mathematical structure

- About $x=a$, $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots$ when the expansion is valid. At $a=0$ it is a Maclaurin expansion. Finite sums may also simplify by cancellation or standard sum formulae.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

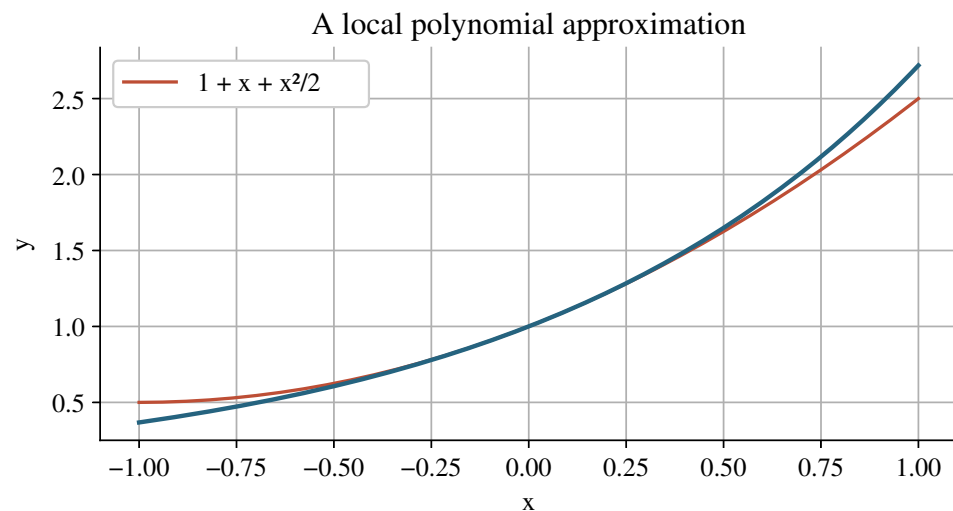
■ Work through a checked case

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

The first three terms of e^x are $1+x+\frac{x^2}{2}$. At $x=0.1$ this gives 1.105, close to $e^{0.1} \approx 1.105170$. For the sum of k^2 from $k=1$ to 5, $n(n+1)(2n+1)/6$ gives $5 \times 6 \times 11 / 6 = 55$.



Key:

A polynomial formed from a function's derivatives at a chosen centre. Choose the relationship, show the method, check its assumptions and interpret the result.

■ Test a tempting shortcut

- The factorial belongs in the denominator of every Taylor coefficient. The expansion centre is not always zero. An approximation close to its centre can be poor far away.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A Taylor polynomial is always exact for the original function at every input. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For a telescoping series, write several terms and identify both surviving ends. For a Taylor approximation, state its order and compare with a known value or a remainder estimate where required.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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