Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

replace x by $2x$; $\mathbf{1 + 2x + 2x^2}$

2.2

$f(0) = 0$, $f'(0) = 1$, $f''(0) = -1$, $f^{(3)}(0) = 2$; $\mathbf{x - \frac{x^2}{2} + \frac{x^3}{3}}$

2.3

$\sin x - x = -\frac{x^3}{6} + \frac{x^5}{120} - \cdots$; $\dfrac{\sin x - x}{x^3} = -\frac{1}{6} + \frac{x^2}{120} - \cdots$; limit $\mathbf{-\frac{1}{6}}$

3.1

(a) $\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(x - \frac{x^3}{6}\right)$; $x + x^2 + \frac{x^3}{2} - \frac{x^3}{6} = x + x^2 + \frac{1}{3}x^3$

(b) $\left[\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{12}\right]_0^{0.2} = \mathbf{0.0228}$

(c) x is small (at most 0.2) (R1), so the missing terms in x^4 and above are very small (R1)

3.2

(a) $1 + \frac{1}{2}x + \dfrac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}x^2 = \mathbf{1 + \frac{x}{2} - \frac{x^2}{8}}$

(b) $x = 0.1$; $1 + 0.05 - 0.00125 = \mathbf{1.04875}$

(c) $\dfrac{|1.04875 - 1.048809|}{1.048809} \times 100 = \mathbf{0.0056}\%$

(d) $\sqrt{3}$ needs $x = 2$; the series is valid only for $|x| < 1$ (R1)

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