Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\dfrac{(x+h)^2 - x^2}{h} = \dfrac{2xh + h^2}{h} = 2x + h$; as $h \to 0$, $f'(x) = \mathbf{2x}$

2.2

$2x + y + x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 0$; $4 + 5\dfrac{dy}{dx} = 0$, $\dfrac{dy}{dx} = -\frac{\mathbf{4}}{\mathbf{5}}$

2.3

$\frac{0}{0}$, so $\lim \dfrac{e^x - 1}{2x}$; still $\frac{0}{0}$: $\lim \dfrac{e^x}{2} = \frac{\mathbf{1}}{\mathbf{2}}$

3.1

(a) $\dfrac{dV}{dr} = 4\pi r^2$; $50 = 4\pi(10)^2\dfrac{dr}{dt}$; $\dfrac{dr}{dt} = \frac{1}{8\pi} = \mathbf{0.0398}$ cm s^{-1}

(b) $\dfrac{dS}{dr} = 8\pi r$; $\dfrac{dS}{dt} = 8\pi(10) \times \frac{1}{8\pi} = \mathbf{10}$ cm^2 s^{-1}

3.2

(a) $\dfrac{1}{1 + (2x)^2} \times 2 = \dfrac{\mathbf{2}}{\mathbf{1 + 4x^2}}$

(b) $\mathbf{3^x \ln 3}$

(c) $\cos y\dfrac{dy}{dx} = 1$; $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$, positive because $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (R1)

(d) $-(\cos x)^{-2}(-\sin x) = \sec \mathbf{x} \tan \mathbf{x}$