Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\int_0^3 kx^2\,dx = 9k = 1;\ k = \mathbf{\frac{1}{9}}$

2.2

$E(X) = \int_0^3 \frac{1}{9}x^3\,dx = \mathbf{\frac{9}{4}};\ E(X^2) = \frac{27}{5};\ \mathrm{Var}(X) = \frac{27}{5} - \frac{81}{16} = \mathbf{0.3375}$

2.3

$\int_0^m \frac{1}{9}x^2\,dx = \frac{1}{2};\ \frac{m^3}{27} = \frac{1}{2};\ m = \mathbf{2.38}$

3.1

(a) 0.02, 0.98; 0.95, 0.05 and 0.08, 0.92

(b) $0.02 \times 0.95 + 0.98 \times 0.08 = \mathbf{0.0974}$

(c) $\dfrac{0.019}{0.0974} = \mathbf{0.195}$

(d) most positive results are false positives, so a positive result needs a second test (R1)

3.2

(a) $\int_0^4 (4x - x^2)\,dx = 32 - \frac{64}{3} = \frac{32}{3};\ k \times \frac{32}{3} = 1,\ k = \frac{3}{32}$

(b) the graph is symmetric about $x = 2$: mode = mean = **2**

(c) $E(X^2) = \frac{3}{32}\int_0^4 (4x^3 - x^4)\,dx = \frac{24}{5};\ \mathrm{Var}(X) = \frac{24}{5} - 4 = \mathbf{0.8}$

(d) $9 \times 0.8 = \mathbf{7.2}$