

## 2.2 Exponentials, logarithms and modelling

---

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 13 marks**

**KEY WORDS** half-life

### Objective

---

Exponentials, logarithms and modelling. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

### 1 Key knowledge and worked examples

---

■ **Why does a decay model stay positive?**

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
  - This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.
- 

■ **Choose the mathematical structure**

- For  $y = Ae^{kt}$ ,  $k$  is a proportional rate. Taking logs gives  $\ln y = \ln A + kt$ . Logarithms require positive arguments, and  $\log(x+y)$  is not  $\log x + \log y$ .
  - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
- 

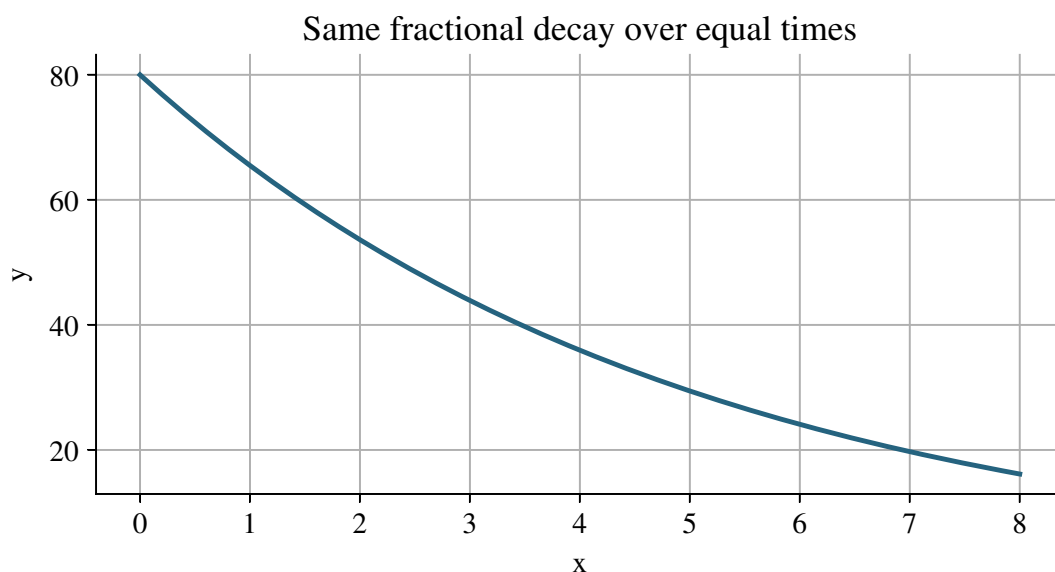
■ **Work through a checked case**

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

**Example:**

If  $y = 80e^{(-0.2t)}$ ,  $y = 40$  gives  $e^{(-0.2t)} = 1/2$ . Hence  $t = \ln 2 / 0.2 \approx 3.466$ . For  $3^x = 20$ ,  $x = \ln 20 / \ln 3$ . Plotting  $\ln y$  against  $t$  linearises this exponential model.



### ■ Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative  $k$  describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

### Warn:

For positive  $x$  and  $y$ ,  $\ln(x+y) = \ln x + \ln y$ . This claim is false. Explain which definition or assumption it violates.

### ■ Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

**Key:**

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.

## 2 Practice

---

**2.1** Solve  $2^x = 32$ . Show the relation, substitution and units. [2]

---

---

---

**2.2** Find  $\ln(e^3)$ . Show the relation, substitution and units. [2]

---

---

---

**2.3** Find the half-life of  $80e^{(-0.2t)}$ , to three decimal places. Show the relation, substitution and units. [2]

---

---

---

**2.4** Decide whether this claim is true and justify: For positive  $x$  and  $y$ ,  $\ln(x+y) = \ln x + \ln y$ . [2]

---

---

---

**2.5** Which description correctly defines half-life? Choose one answer.

- **A** The time for a decaying quantity to fall to half its initial value.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

---

### 3 Exam-style questions

---

**3.1** A fictional quantity follows  $80 \exp(-0.2 t)$ . Find when it reaches 20, retaining an exact expression; explain why the half-life does not depend on 80. [4]

---

---

---

---

---

---

---

---

---

---