Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

e.g. $n = 40$: $40^2 + 40 + 41 = 1681 = 41^2$, not prime

2.2

assume n is odd, $n = 2k + 1$; $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, which is odd; this contradicts n^2 even, so n is even (R1)

3.1

$n = 1$: $1 \times 2 = 2$ and $\frac{1 \times 2 \times 3}{3} = 2$; assume true for $n = k$; $\sum_{r=1}^{k+1} r(r+1) = \frac{k(k+1)(k+2)}{3} +$ $(k + 1)(k + 2)$; $= \frac{(k+1)(k+2)(k+3)}{3}$; true for $n = 1$ and true for k implies true for $k + 1$, so true for all $n \in \mathbb{Z}^+$ (R1)

3.2

$n = 1$: $7 - 1 = 6$; assume $7^k - 1 = 6m$, $m \in \mathbb{Z}$; $7^{k+1} - 1 = 7(6m + 1) - 1$; $= 42m + 6 = 6(7m + 1)$; so true for all $n \in \mathbb{Z}^+$ by induction (R1)