

5.3 Chain, product, quotient and implicit differentiation

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS chain rule

Objective

Chain, product, quotient and implicit differentiation. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** : The rule that multiplies the outer derivative by the inner derivative for a composite function.
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■ Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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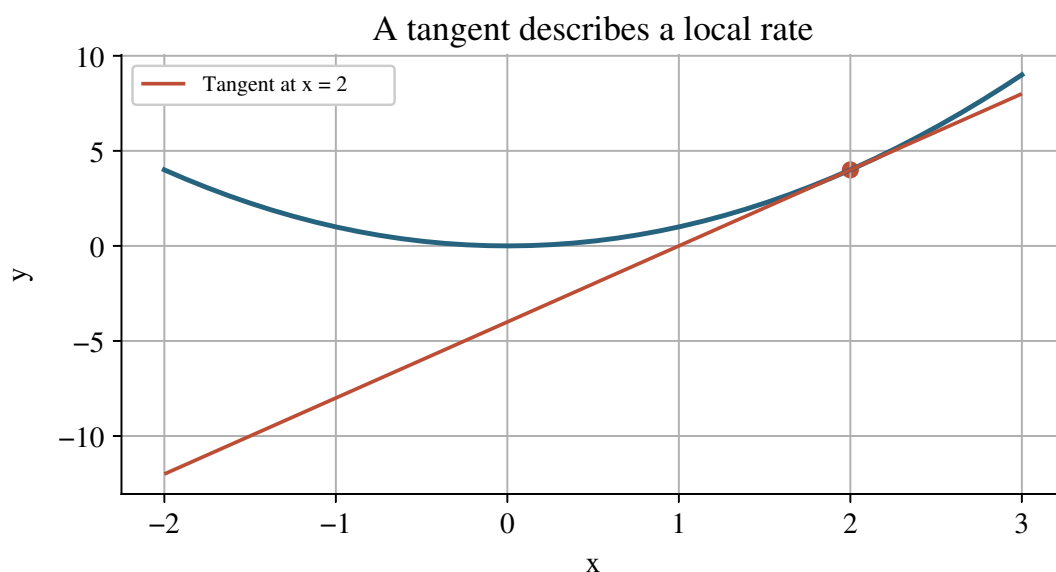
■ Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



■ **Test a tempting shortcut**

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the derivative of $(3x+1)^4$ at $x=0$. Show the relation, substitution and units. [2]

2.2 For $x^2+y^2=25$, find dy/dx at $(3,4)$. Show the relation, substitution and units. [2]

2.3 For $y=xe^x$, find dy/dx at $x=0$. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: The derivative of $u(x)v(x)$ is always u prime times v prime. [2]

2.5 Which description correctly defines chain rule? Choose one answer.

- **A** The rule that multiplies the outer derivative by the inner derivative for a composite function.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 For $x^2 + y^2 = 25$, find dy/dx and discuss its values at $(3, 4)$ and $(5, 0)$, using geometry to check.

[4]

Solutions

2.1 Chain rule gives $4(3x+1)^3 \times 3$; at zero this is 12.

2.2 Implicit differentiation gives $2x+2y y' = 0$, so $y' = -3/4$.

2.3 Product rule gives $e^x + xe^x$; at zero this is 1.

2.4 False. A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.

2.5 A The rule that multiplies the outer derivative by the inner derivative for a composite function.

3.1 $y' = -x/y$ where $y \neq 0$. At $(3, 4)$, $-3/4$. At $(5, 0)$ the tangent is vertical, so finite dy/dx is undefined; implicit differentiation does not license division by zero.