

5.1 Derivatives and stationary points

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS derivative

Objective

Derivatives and stationary points. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.

■ Choose the mathematical structure

- For $y = ax^n$, $dy/dx = anx^{n-1}$. A stationary point satisfies $dy/dx = 0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

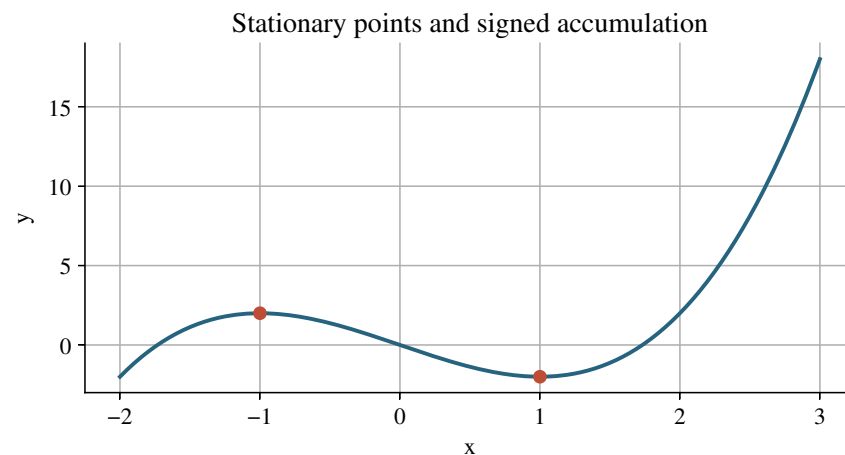
■ Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $y = x^3 - 3x$, $dy/dx = 3x^2 - 3$. At $x = 1$, the gradient is 0 and $y = -2$. The second derivative is $6x$, positive at $x = 1$, so this is a local minimum. At $x = -1$, $y = 2$ and the second derivative is negative, giving a local maximum.



■ Test a tempting shortcut

- A zero derivative does not always mean a maximum or minimum: $y = x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 For $y=x^3-3x$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.2 Find the y-coordinate of the stationary point at $x=1$. Show the relation, substitution and units. [2]

2.3 For $y=4x^3$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Every point with zero derivative is a local maximum or minimum. [2]

2.5 Which description correctly defines derivative? Choose one answer.

- **A** The instantaneous rate of change, also the gradient of a tangent.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 For $y=x^3-3x$ on $[-2, 2]$, find the absolute maximum and minimum, including all candidates. [4]

Solutions

2.1 $dy/dx=3x^2-3$. At $x=2$ it is $12-3=9$.

2.2 Substitute into the original curve: $y(1)=1-3=-2$.

2.3 The derivative is $12x^2$; at 2 it is $12 \times 4=48$.

2.4 False. A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.

2.5 A The instantaneous rate of change, also the gradient of a tangent.

3.1 Derivative $3x^2-3=0$ at ± 1 . Values $y(-2)=-2$, $y(-1)=2$, $y(1)=-2$, $y(2)=2$. Maximum 2 at -1 , 2; minimum -2 at -2 , 1. Checking only stationary points misses endpoint ties.