

## 4.3 Binomial, normal and Poisson models

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 13 marks

**KEY WORDS** expected value

### Objective

Binomial, normal and Poisson models. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

### 1 Key knowledge and worked examples

#### ■ What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** : The probability-weighted mean of a random variable.

#### ■ Choose the mathematical structure

- A binomial model needs fixed  $n$ , independent trials, two outcomes and constant  $p$ .  $E(X)=np$  and  $\text{Var}(X)=np(1-p)$ . For a normal model use  $z=(x-\mu)/\sigma$ . A Poisson model describes counts with a constant rate and appropriate independence assumptions.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

#### ■ Work through a checked case

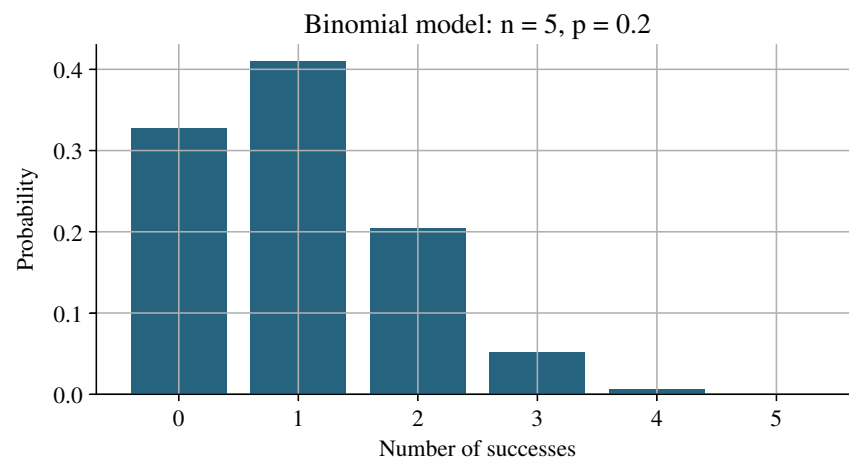
$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad X \sim B(n, p)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

#### Example:

For  $X$  binomial(5,0.2),  $P(X=0)=0.8^5=0.32768$ ,  $E(X)=1$  and  $\text{Var}(X)=0.8$ . For a normal quantity with  $\mu=100, \sigma=15$ , the value 130 has  $z=2$ . A Poisson mean of 3 per hour gives

mean 6 over two hours.



#### ■ Test a tempting shortcut

- Not every count is binomial: changing  $p$  or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

#### Warn:

Every count of successes has a binomial distribution regardless of dependence. This claim is false. Explain which definition or assumption it violates.

#### ■ Interpret a new situation

- Write the event as an inequality before using calculator distribution functions. Distinguish  $P(X < k)$ ,  $P(X \leq k)$  and a tail complement. State assumptions in context.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

#### ■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

#### Key:

The probability-weighted mean of a random variable. Choose the relationship, show the method, check its assumptions and interpret the result.

## 2 Practice

**2.1** For  $X \sim \text{binomial}(5, 0.2)$ , find  $P(X=0)$ . Show the relation, substitution and units. [2]

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**2.2** Find  $\text{Var}(X)$  for  $X \sim \text{binomial}(5, 0.2)$ . Show the relation, substitution and units. [2]

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**2.3** For  $\mu=100$  and  $\sigma=15$ , find the z-score of 130. Show the relation, substitution and units. [2]

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**2.4** Decide whether this claim is true and justify: Every count of successes has a binomial distribution regardless of dependence. [2]

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**2.5** Which description correctly defines expected value? Choose one answer.

- **A** The probability-weighted mean of a random variable.
  - **B** A numerical answer that needs no assumptions or interpretation
  - **C** A rule that remains valid even when its required conditions fail
- [1]

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## 3 Exam-style questions

**3.1** For  $X \sim \text{Binomial}(5, 0.2)$ , find  $P(X \geq 1)$  and the mean. Explain one situation in which the binomial model would be inappropriate. [4]

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## Solutions

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**2.1** No successes means five failures:  $0.8^5=0.32768$ .

**2.2** Binomial variance= $np(1-p)=5\times 0.2\times 0.8=0.8$ .

**2.3** Standardise:  $z=(130-100)/15=2$ .

**2.4** False. Not every count is binomial: changing  $p$  or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.

**2.5 A** The probability-weighted mean of a random variable.

**3.1**  $1-0.8^5=0.67232$ ; mean  $np=1$ . Trials must be independent with constant success probability and fixed count; depletion or correlated outcomes can violate this.