

4.3 Binomial, normal and Poisson models

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS expected value

Objective

Binomial, normal and Poisson models. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
 - This lesson studies **expected value** : The probability-weighted mean of a random variable.
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■ Choose the mathematical structure

- A binomial model needs fixed n , independent trials, two outcomes and constant p . $E(X)=np$ and $\text{Var}(X)=np(1-p)$. For a normal model use $z=(x-\mu)/\sigma$. A Poisson model describes counts with a constant rate and appropriate independence assumptions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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■ Work through a checked case

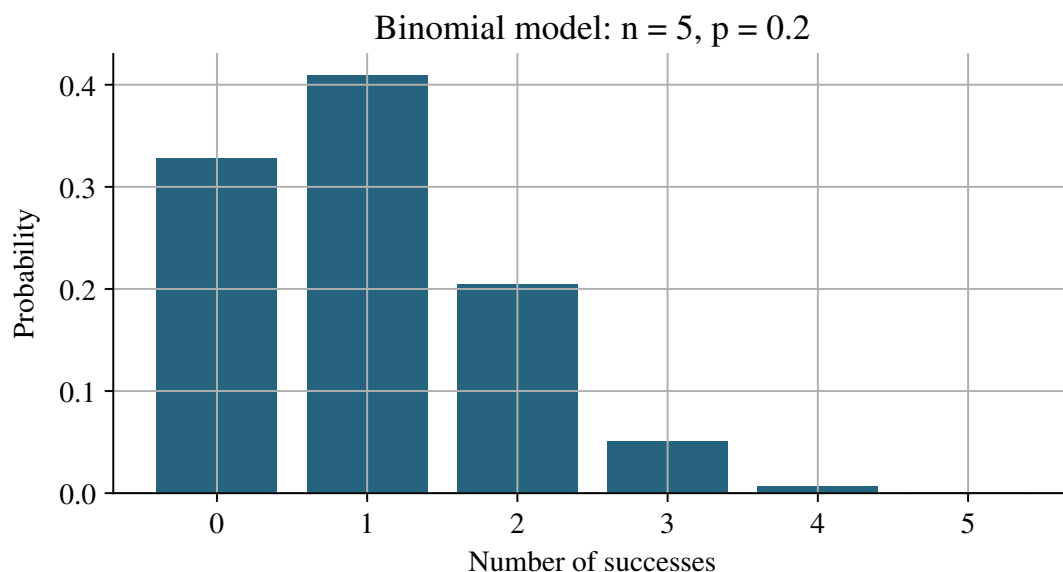
$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad X \sim B(n, p)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For X binomial(5,0.2), $P(X=0)=0.8^5=0.32768$, $E(X)=1$ and $\text{Var}(X)=0.8$. For a normal quantity with $\mu=100, \sigma=15$, the value 130 has $z=2$. A Poisson mean of 3 per hour gives

mean 6 over two hours.



■ Test a tempting shortcut

- Not every count is binomial: changing p or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every count of successes has a binomial distribution regardless of dependence. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Write the event as an inequality before using calculator distribution functions. Distinguish $P(X < k)$, $P(X \leq k)$ and a tail complement. State assumptions in context.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The probability-weighted mean of a random variable. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 For X binomial(5,0.2), find $P(X=0)$. Show the relation, substitution and units. [2]

2.2 Find $\text{Var}(X)$ for X binomial(5,0.2). Show the relation, substitution and units. [2]

2.3 For $\mu=100$ and $\sigma=15$, find the z-score of 130. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Every count of successes has a binomial distribution regardless of dependence. [2]

2.5 Which description correctly defines expected value? Choose one answer.

- **A** The probability-weighted mean of a random variable.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 For $X \sim \text{Binomial}(5, 0.2)$, find $P(X \geq 1)$ and the mean. Explain one situation in which the binomial model would be inappropriate. [4]

Solutions

2.1 No successes means five failures: $0.8^5=0.32768$.

2.2 Binomial variance= $np(1-p)=5\times 0.2\times 0.8=0.8$.

2.3 Standardise: $z=(130-100)/15=2$.

2.4 False. Not every count is binomial: changing p or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.

2.5 A The probability-weighted mean of a random variable.

3.1 $1-0.8^5=0.67232$; mean $np=1$. Trials must be independent with constant success probability and fixed count; depletion or correlated outcomes can violate this.