

3.4 Vectors and transformation geometry

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS resultant

Objective

Vectors and transformation geometry. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ **Why is displacement shorter than the walk?**

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
 - This lesson studies **resultant** : The vector sum representing the combined displacement or force.
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■ **Choose the mathematical structure**

- Add corresponding vector components. A displacement from A to B is OB-OA. Parallel vectors are scalar multiples. A vector has magnitude and direction; a distance is a scalar.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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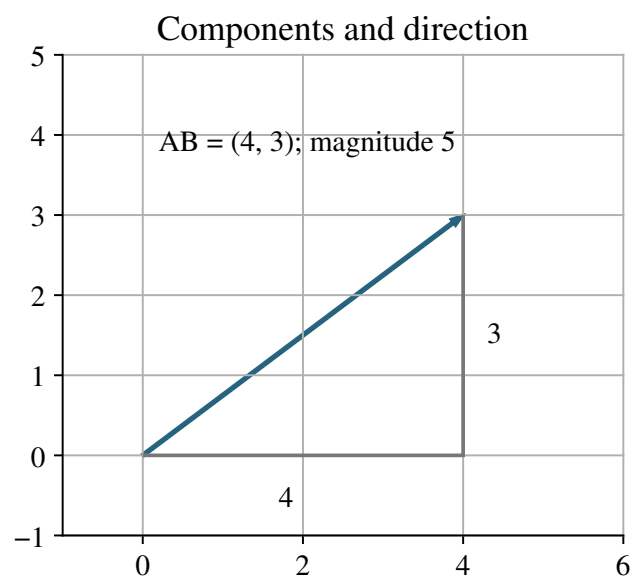
■ **Work through a checked case**

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. Their scalar product is $\mathbf{a} \cdot \mathbf{b}=4 \times 1+1 \times 3=7$. The magnitude of $\mathbf{a}$ is $\sqrt{17}$. For $\mathbf{A}=(1,2)$, $\mathbf{B}=(5,5)$, $\mathbf{AB}=(4,3)$ and $|\mathbf{AB}|=5$.



■ **Test a tempting shortcut**

- The order of subtraction matters: $\vec{BA} = -\vec{AB}$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

$\vec{AB}$ and $\vec{BA}$ always have the same components. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- For a translation, every point uses the same displacement. For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the x-component of $(4,1)+(1,3)$. Show the relation, substitution and units. [2]

2.2 Find the magnitude of $(4,3)$. Show the relation, substitution and units. [2]

2.3 Find the scalar product of $(4,1)$ and $(1,3)$. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: $\overrightarrow{AB}$ and $\overrightarrow{BA}$ always have the same components. [2]

2.5 Which description correctly defines resultant? Choose one answer.

- **A** The vector sum representing the combined displacement or force.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 Points $A=(1, 2, 0)$, $B=(3, 2, 1)$, $C=(1, 5, 0)$. Find $\overrightarrow{AB} \cdot \overrightarrow{AC}$ and decide whether the angle at A is right; distinguish a zero vector exception.

[4]

Solutions

2.1 Add the x-components: $4+1=5$.

2.2 Magnitude= $\sqrt{4^2+3^2}=5$.

2.3 Multiply corresponding components and add: $4\times 1+1\times 3=7$.

2.4 False. The order of subtraction matters: $\mathbf{BA}=-\mathbf{AB}$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.

2.5 A The vector sum representing the combined displacement or force.

3.1 $\mathbf{AB}=(2, 0, 1)$, $\mathbf{AC}=(0, 3, 0)$, dot 0. Both nonzero, so angle $\pi/2$. A zero dot product involving a zero vector would not define an angle.