

3.3 Radians, identities and trigonometric equations

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS radian

Objective

Radians, identities and trigonometric equations. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.

■ Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=\frac{1}{2}r^2\theta$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

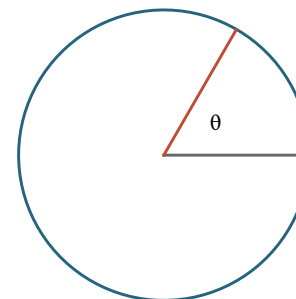
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

■ Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 For $r=4$ and $\theta=0.5$ radians, find arc length. Show the relation, substitution and units. [2]

2.2 For $r=4$ and $\theta=0.5$ radians, find sector area. Show the relation, substitution and units. [2]

2.3 How many solutions does $\sin x=0.5$ have on $0 \leq x < 2\pi$? Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: The principal inverse sine value is always the only solution in a full turn. [2]

2.5 Which description correctly defines radian? Choose one answer.

- **A** The angle subtended by an arc whose length equals the radius.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 Solve $2 \sin^2 x - \sin x - 1 = 0$ for $0 \leq x < 2\pi$, showing why no solutions are lost. [4]

Solutions

2.1 Arc length= $r\theta=4\times 0.5=2$.

2.2 Sector area= $r^2\theta/2=16\times 0.5/2=4$.

2.3 The two solutions are $\pi/6$ and $5\pi/6$ in the stated interval.

2.4 False. A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.

2.5 A The angle subtended by an arc whose length equals the radius.

3.1 Factor $(2 \sin x + 1)(\sin x - 1) = 0$. $\sin x = 1$ gives $\pi/2$; $\sin x = -1/2$ gives $7\pi/6, 11\pi/6$. Both allowed roots are considered and the endpoint interval is respected.