

3.1 Angle reasoning, similarity and mensuration

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS scale factor

Objective

Angle reasoning, similarity and mensuration. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

■ Choose the mathematical structure

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 . State angle reasons explicitly. A circle's tangent is perpendicular to the radius at the contact point.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

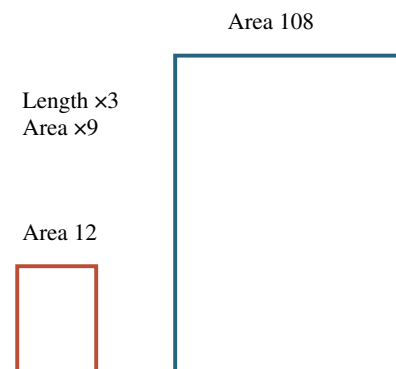
- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

If model-to-real length factor is 3, a model area of 12 cm² gives $12 \times 3^2 = 108$ cm² and a model volume of 8 cm³ gives $8 \times 3^3 = 216$ cm³. A cylinder with $r=3, h=5$ has volume

$$\pi r^2 h = 45\pi.$$

A length scale factor is squared for area



■ Test a tempting shortcut

- Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Doubling every length of a solid doubles its volume. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion. A scale drawing is evidence only when the task permits measurement.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The multiplier that relates corresponding lengths in similar shapes. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

2.2 Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

2.3 Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Doubling every length of a solid doubles its volume. [2]

2.5 Which description correctly defines scale factor? Choose one answer.

- **A** The multiplier that relates corresponding lengths in similar shapes.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

3 Exam-style questions

3.1 Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

Solutions

2.1 Area scales by the square of the length factor: $3^2=9$.

2.2 Volume scales by the cube: $3^3=27$.

2.3 Interior-angle sum= $(5-2)\times 180=540^\circ$.

2.4 False. Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

2.5 A The multiplier that relates corresponding lengths in similar shapes.

3.1 Length ratio 3:2, area 9:4. Area scales as the square and volume as the cube of the length factor, assuming genuine geometric similarity.