

1.4 Percentages, ratio and proportional reasoning

Name: _____ Class: _____ Date: _____ Total: 13 marks

KEY WORDS multiplier

Objective

Percentages, ratio and proportional reasoning. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier** : A factor that performs a percentage change in one multiplication.

■ Choose the mathematical structure

- A p% increase has multiplier $1 + p/100$; a decrease has multiplier $1 - p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

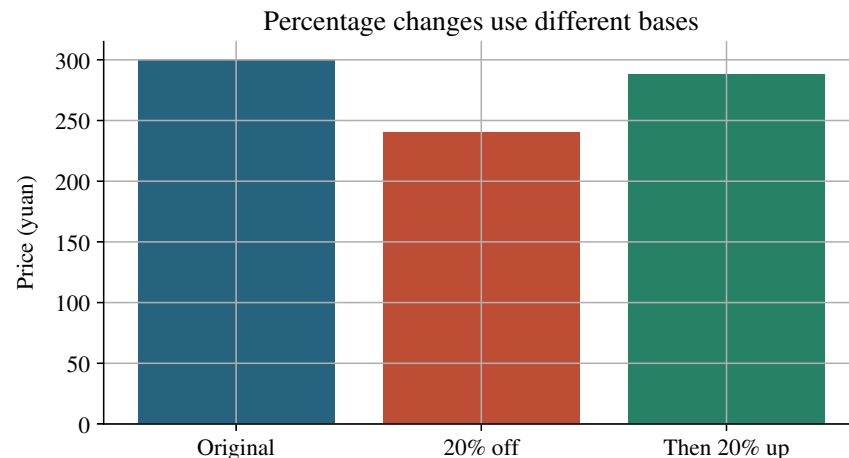
■ Work through a checked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

Let the original price be P. The model is sale price = $0.8P$. Hence $P = 240/0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



■ Test a tempting shortcut

- A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A 20% decrease followed by a 20% increase restores the starting price. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For direct proportion use $y = kx$; for inverse proportion use $y = k/x$. Calculate k from a known pair before using a new value. State what you held constant.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A factor that performs a percentage change in one multiplication. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 A price of 80 rises by 15%. Find the new price. Show the relation, substitution and units. [2]

2.2 A 25% discount leaves a price of 90. Find the original price. Show the relation, substitution and units. [2]

2.3 Share 84 in the ratio 2:5. Find the larger share. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: A 20% decrease followed by a 20% increase restores the starting price. [2]

2.5 Which description correctly defines multiplier? Choose one answer.

- **A** A factor that performs a percentage change in one multiplication.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 A price rises 20% then falls 20%. Find the net change and the percentage increase needed to recover the original after that pair. [4]

Solutions

2.1 Multiply by 1.15: $80 \times 1.15 = 92$.

2.2 The sale price is 75% of the original: $90 / 0.75 = 120$.

2.3 There are seven ratio parts. The larger share is $84 \times 5/7 = 60$.

2.4 False. A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.

2.5 A A factor that performs a percentage change in one multiplication.

3.1 Factor $1.2 \times 0.8 = 0.96$, a 4% fall. Recovery factor $1/0.96$, giving 4.1667% increase. Both percentage changes use different bases.