

1.3 Sequences, series and recurrence

Name: _____ Class: _____ Date: _____ Total: 13 marks

KEY WORDS common ratio

Objective

Sequences, series and recurrence. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** : The constant multiplier between consecutive terms of a geometric sequence.

■ Choose the mathematical structure

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

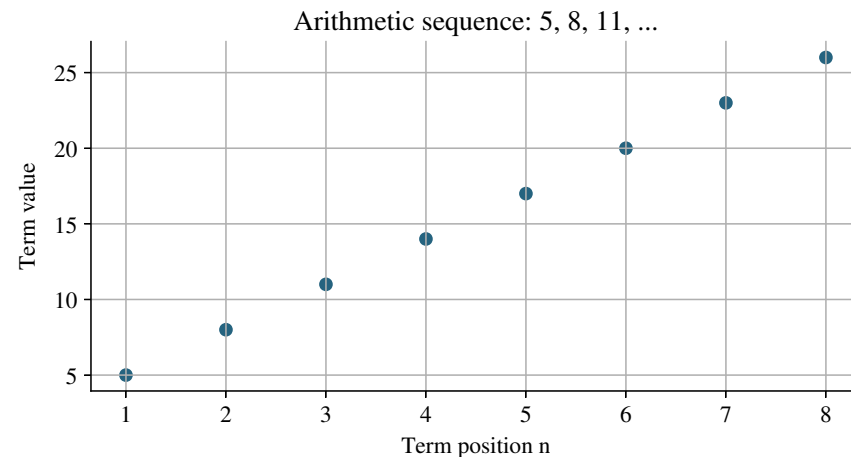
■ Work through a checked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10 + 21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



■ Test a tempting shortcut

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 For $a=5, d=3$, find u_8 . Show the relation, substitution and units. [2]

2.2 Find the sum of the first 8 terms with $a=5, d=3$. Show the relation, substitution and units. [2]

2.3 Find the infinite sum with $a=12, r=0.5$. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Every geometric series has a finite sum to infinity. [2]

2.5 Which description correctly defines common ratio? Choose one answer.

- **A** The constant multiplier between consecutive terms of a geometric sequence.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 A sequence obeys $u_1=4$ and $u_{n+1}=0.5 u_n+3$. Find its fixed point and express u_n-6 as a geometric sequence. Does the sum of u_n converge? [4]

Solutions

2.1 $u_8 = 5 + (8-1) \times 3 = 26$.

2.2 $S_8 = 8[2 \times 5 + 7 \times 3] / 2 = 124$.

2.3 Since $|r| < 1$, $S_\infty = 12 / (1 - 0.5) = 24$.

2.4 False. The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.

2.5 A The constant multiplier between consecutive terms of a geometric sequence.

3.1 Fixed point 6. Subtracting 6 gives $u_{n+1} - 6 = 0.5(u_n - 6)$, so $u_n = 6 - 2(0.5)^{n-1}$. Terms approach 6, so their infinite sum diverges; a convergent sequence need not give a convergent series.