

1.2 Indices, surds and standard form

Name: _____ Class: _____ Date: _____ Total: 13 marks

KEY WORDS index

Objective

Indices, surds and standard form. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

■ Choose the mathematical structure

- For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \leq a < 10$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

■ Test a tempting shortcut

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For every positive a , $a^2 + a^3$ equals a^5 . This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The power to which a base is raised. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Evaluate $16^{3/4}$. Show the relation, substitution and units. [2]

2.2 Evaluate 2^{-3} . Show the relation, substitution and units. [2]

2.3 Write 7.2×10^{-5} as a decimal. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: For every positive a , $a^2 + a^3$ equals a^5 . [2]

2.5 Which description correctly defines index? Choose one answer.

- **A** The power to which a base is raised.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 Solve $x^{3/2} = 27$ over the real domain used for this power. Check the result and explain why a negative candidate is invalid. [4]

Solutions

2.1 $16=2^4$, so $16^{(3/4)}=2^3=8$.

2.2 A negative index gives a reciprocal: $2^{(-3)}=1/8=0.125$.

2.3 Move the decimal point five places left: $7.2 \times 10^{(-5)}=0.000072$.

2.4 False. Index laws do not turn a sum into a single power: $2^3+2^4=24$, not 2^7 . Do not round a surd when an exact answer is requested.

2.5 A The power to which a base is raised.

3.1 Take the nonnegative real square-root convention: $x \geq 0$. $x=27^{(2/3)}=9$; $9^{(3/2)}=27$. A negative x has no real square root under this convention.