

Exercise walk-through

# 4.2

## Probability, trees and conditional reasoning

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International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

# 4.2 Probability, trees and conditional reasoning

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 13 marks**

**KEY WORDS** conditional probability

# Objective

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Probability, trees and conditional reasoning. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

## 1 Key knowledge and worked examples

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### ■ What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

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■ **Choose the mathematical structure**

- Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is  $P(A \text{ given } B) = P(A \cap B) / P(B)$ , for  $P(B) > 0$ .
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

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■ **Work through a checked case**

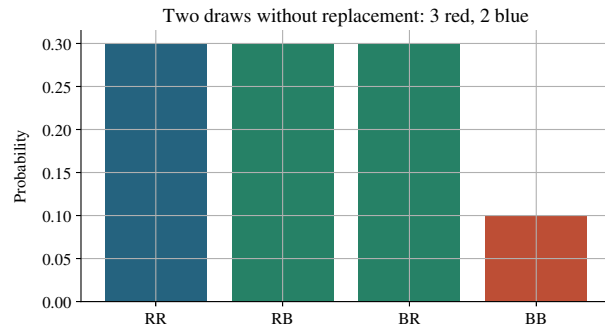
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

**Example:**

Without replacement,  $P(\text{two red}) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$ .  $P(\text{one of each}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$ .  
If  $P(A \cap B) = 0.12$  and  $P(B) = 0.3$ ,  $P(A \text{ given } B) = 0.4$ .

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■ **Test a tempting shortcut**

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

**Warn:**

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

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■ **Interpret a new situation**

- A two-way table makes the restricted denominator visible. Before using a product  $P(A)P(B)$ , justify independence from the context or the supplied information.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

**Key:**

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

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## Exercises 2.1-2.3

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**2.1** Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

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**2.2** Find  $P(A \text{ given } B)$  if  $P(A \cap B) = 0.12$  and  $P(B) = 0.3$ . Show the relation, substitution and units. [2]

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**2.3** Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

## Exercises 2.1·2.3

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**2.1** Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

**2.1** Without replacement,  $P(RR)=3/5 \times 2/4=0.3$ .

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**2.2** Find  $P(A \text{ given } B)$  if  $P(A \cap B)=0.12$  and  $P(B)=0.3$ . Show the relation, substitution and units. [2]

**2.2**  $P(A|B)=P(A \cap B)/P(B)=0.12/0.3=0.4$ .

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**2.3** Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

**2.3** Add the two orders:  $3/5 \times 2/4 + 2/5 \times 3/4 = 0.6$ .

## Exercises 2.4·2.5

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**2.4** Decide whether this claim is true and justify: Mutually exclusive events with positive probabilities must be independent. [2]

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**2.5** Which description correctly defines conditional probability? Choose one answer.

- **A** The probability of an event after restricting the sample space to a stated condition.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

## 3 Exam-style questions

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## Exercises 2.4·2.5

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**2.4** Decide whether this claim is true and justify: Mutually exclusive events with positive probabilities must be independent. [2]

**2.4** False. Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.

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## 3 Exam-style questions

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**2.5 A** The probability of an event after restricting the sample space to a stated condition.

## Exercise 3.1

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**3.1** A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

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**3.1** A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

**3.1**  $P(RR) = \frac{3}{5} \times \frac{2}{4} = 0.3$ .  $P(\text{at least one } R) = 1 - \frac{2}{5} \times \frac{1}{4} = 0.9$ . Conditional probability  $= 0.3 / 0.9 = 1/3$ . The conditioning changes the sample space.