

Exercise walk-through

3.4

Vectors and transformation geometry

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

3.4 Vectors and transformation geometry

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS resultant

Objective

Vectors and transformation geometry. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** : The vector sum representing the combined displacement or force.

■ **Choose the mathematical structure**

- Add corresponding vector components. A displacement from A to B is OB-OA. Parallel vectors are scalar multiples. A vector has magnitude and direction; a distance is a scalar.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

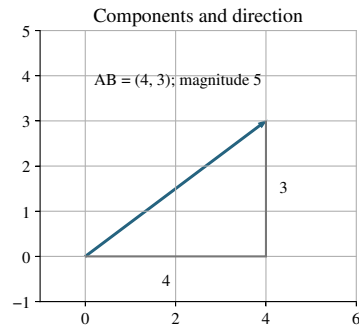
■ **Work through a checked case**

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

With $a=(4,1)$ and $b=(1,3)$, $a+b=(5,4)$. Their scalar product is $a \cdot b=4 \times 1+1 \times 3=7$. The magnitude of a is $\sqrt{17}$. For $A=(1,2)$, $B=(5,5)$, $AB=(4,3)$ and $|AB|=5$.



■ **Test a tempting shortcut**

- The order of subtraction matters: $BA = -AB$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

AB and BA always have the same components. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- For a translation, every point uses the same displacement. For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 Find the x-component of $(4,1)+(1,3)$. Show the relation, substitution and units. [2]

2.2 Find the magnitude of $(4,3)$. Show the relation, substitution and units. [2]

2.3 Find the scalar product of $(4,1)$ and $(1,3)$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find the x-component of $(4,1)+(1,3)$. Show the relation, substitution and units. [2]

2.1 Add the x-components: $4+1=5$.

2.2 Find the magnitude of $(4,3)$. Show the relation, substitution and units. [2]

2.2 Magnitude= $\sqrt{4^2+3^2}=5$.

2.3 Find the scalar product of $(4,1)$ and $(1,3)$. Show the relation, substitution and units. [2]

2.3 Multiply corresponding components and add: $4\times 1+1\times 3=7$.

Exercises 2.4-3.1

2.4 Decide whether this claim is true and justify: AB and BA always have the same components. [2]

2.5 Which description correctly defines resultant? Choose one answer.

- **A** The vector sum representing the combined displacement or force.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3.1 Points $A=(1, 2, 0)$, $B=(3, 2, 1)$, $C=(1, 5, 0)$. Find $AB \cdot AC$ and decide whether the angle at A is right; distinguish a zero vector exception. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: AB and BA always have the same components. [2]

2.4 False. The order of subtraction matters: $BA = -AB$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.

2.5 Which description correctly defines resultant? Choose one answer.

- **A** The vector sum representing the combined displacement or force.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

2.5 A The vector sum representing the combined displacement or force.

3.1 Points $A=(1, 2, 0)$, $B=(3, 2, 1)$, $C=(1, 5, 0)$. Find $AB \cdot AC$ and decide whether the angle at A is right; distinguish a zero vector exception. [4]

3.1 $AB=(2, 0, 1)$, $AC=(0, 3, 0)$, dot 0. Both nonzero, so angle $\pi/2$. A zero dot product involving a zero vector would not define an angle.