

Exercise walk-through

3.1

Angle reasoning, similarity and mensuration

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

3.1 Angle reasoning, similarity and mensuration

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS scale factor

Objective

Angle reasoning, similarity and mensuration. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

■ **Choose the mathematical structure**

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 . State angle reasons explicitly. A circle's tangent is perpendicular to the radius at the contact point.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ **Work through a checked case**

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

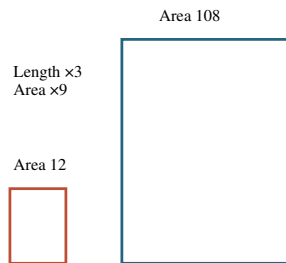
- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume

$$\pi r^2 h = 45\pi.$$

A length scale factor is squared for area



■ **Test a tempting shortcut**

- Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Doubling every length of a solid doubles its volume. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion. A scale drawing is evidence only when the task permits measurement.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The multiplier that relates corresponding lengths in similar shapes. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

2.2 Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

2.3 Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

2.1 Area scales by the square of the length factor: $3^2=9$.

2.2 Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

2.2 Volume scales by the cube: $3^3=27$.

2.3 Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

2.3 Interior-angle sum $= (5-2) \times 180 = 540^\circ$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Doubling every length of a solid doubles its volume. [2]

2.5 Which description correctly defines scale factor? Choose one answer.

- **A** The multiplier that relates corresponding lengths in similar shapes.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

Exercises 2.4-2.5

2.4 Decide whether this claim is true and justify: Doubling every length of a solid doubles its volume. [2]

2.4 False. Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

2.5 Which description correctly defines scale factor? Choose one answer.

- **A** The multiplier that relates corresponding lengths in similar shapes.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

2.5 A The multiplier that relates corresponding lengths in similar shapes.

Exercise 3.1

3.1 Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

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3.1 Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

3.1 Length ratio 3:2, area 9:4. Area scales as the square and volume as the cube of the length factor, assuming genuine geometric similarity.