

Exercise walk-through

2.1

Domains, inverses and composition

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

2.1 Domains, inverses and composition

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS domain

Objective

Domains, inverses and composition. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

■ **Choose the mathematical structure**

- State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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■ **Work through a checked case**

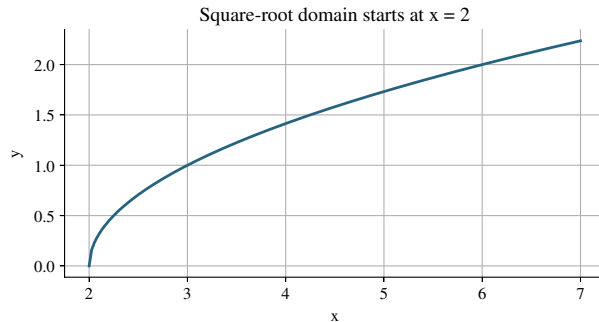
$$f(g(x)) = (f \circ g)(x), \qquad f^{-1}(f(x)) = x$$

- Check the result against the starting quantities. Substitute into the original relation,

or compare the graph and numerical answer where appropriate.

Example:

For $f(x)=\sqrt{x-2}$, $x\geq 2$ and the range is $y\geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus f inverse $(x)=x^2+2$ with $x\geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



■ **Test a tempting shortcut**

- Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every quadratic function on all real numbers has an inverse function. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- Check $f(f^{-1}(x))=x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The set of allowed inputs to a function. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 For $f(x)=2x+3$, find $f^{-1}(11)$. Show the relation, substitution and units. [2]

2.2 For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$. Show the relation, substitution and units. [2]

2.3 For $f(x)=\sqrt{x-2}$, find $f(11)$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $f(x)=2x+3$, find $f^{-1}(11)$. Show the relation, substitution and units. [2]

2.1 Solve $2x+3=11$, giving $x=4$.

2.2 For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$. Show the relation, substitution and units. [2]

2.2 Apply g first: $g(4)=16$; then $f(16)=2 \times 16 + 3 = 35$.

2.3 For $f(x)=\sqrt{x-2}$, find $f(11)$. Show the relation, substitution and units. [2]

2.3 Substitute into the square root: $\sqrt{11-2}=\sqrt{9}=3$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Every quadratic function on all real numbers has an inverse function. [2]

2.5 Which description correctly defines domain? Choose one answer.

- **A** The set of allowed inputs to a function.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 For $f(x)=x^2$ on $x \geq 0$ and $g(x)=x-3$, find $f(g(x))$ and its domain, then find f inverse(16).

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Every quadratic function on all real numbers has an inverse function. [2]

2.4 False. Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.

2.5 Which description correctly defines domain? Choose one answer.

- **A** The set of allowed inputs to a function.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

2.5 A The set of allowed inputs to a function.

3.1 For $f(x)=x^2$ on $x \geq 0$ and $g(x)=x-3$, find $f(g(x))$ and its domain, then find f inverse(16).

3.1 Composite $= (x-3)^2$ with $x \geq 3$ because $g(x)$ must lie in f 's domain. Inverse $16=4$.