

Exercise walk-through

1.3

Sequences, series and recurrence

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

1.3 Sequences, series and recurrence

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS common ratio

Objective

Sequences, series and recurrence. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
 - This lesson studies **common ratio** : The constant multiplier between consecutive terms of a geometric sequence.
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■ Choose the mathematical structure

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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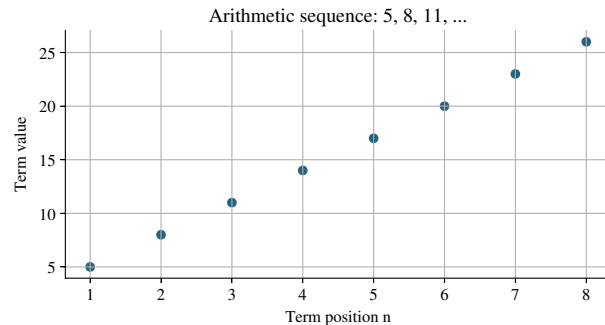
■ Work through a checked case

$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2} [2a + (n - 1)d]$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $a=5, d=3, n=8$, $u_8=5+7\times 3=26$ and $S_8=8(10+21)/2=124$. For $a=12, r=1/2$, $S_{\infty}=12/(1-1/2)=24$. For $u_{(n+1)}=2u_n+1$ with $u_1=1$, the next terms are 3,7,15.



■ **Test a tempting shortcut**

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 For $a=5, d=3$, find u_8 . Show the relation, substitution and units. [2]

2.2 Find the sum of the first 8 terms with $a=5, d=3$. Show the relation, substitution and units. [2]

2.3 Find the infinite sum with $a=12, r=0.5$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $a=5, d=3$, find u_8 . Show the relation, substitution and units. [2]

2.1 $u_8 = 5 + (8-1) \times 3 = 26.$

2.2 Find the sum of the first 8 terms with $a=5, d=3$. Show the relation, substitution and units. [2]

2.2 $S_8 = 8[2 \times 5 + 7 \times 3] / 2 = 124.$

2.3 Find the infinite sum with $a=12, r=0.5$. Show the relation, substitution and units. [2]

2.3 Since $|r| < 1$, $S_\infty = 12 / (1 - 0.5) = 24.$

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every geometric series has a finite sum to infinity. [2]

2.5 Which description correctly defines common ratio? Choose one answer.

- **A** The constant multiplier between consecutive terms of a geometric sequence.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every geometric series has a finite sum to infinity. [2]

2.4 False. The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.

2.5 Which description correctly defines common ratio? Choose one answer.

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- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

2.5 A The constant multiplier between consecutive terms of a geometric sequence.

Exercise 3.1

3.1 A sequence obeys $u_1=4$ and $u_{n+1}=0.5 u_n+3$. Find its fixed point and express u_n-6 as a geometric sequence. Does the sum of u_n converge? [4]

Exercise 3.1

3.1 A sequence obeys $u_1=4$ and $u_{n+1}=0.5 u_n+3$. Find its fixed point and express u_n-6 as a geometric sequence. Does the sum of u_n converge? [4]

3.1 Fixed point 6. Subtracting 6 gives $u_{n+1}-6=0.5(u_n-6)$, so $u_n=6-2(0.5)^{n-1}$. Terms approach 6, so their infinite sum diverges; a convergent sequence need not give a convergent series.