

Exercise walk-through

1.1

Exact arithmetic and estimation

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

1.1 Exact arithmetic and estimation

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS prime factor

Objective

Exact arithmetic and estimation. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

■ Choose the mathematical structure

- Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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■ **Work through a checked case**

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

■ **Test a tempting shortcut**

- The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.3 Work out $\frac{3}{4} + \frac{5}{6}$ as a decimal. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.1 $48=2^4 \times 3$ and $60=2^2 \times 3 \times 5$. The shared smallest powers give $\text{HCF}=2^2 \times 3=12$.

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.2 $8=2^3$ and $12=2^2 \times 3$. The largest prime powers give $\text{LCM}=2^3 \times 3=24$.

2.3 Work out $\frac{3}{4} + \frac{5}{6}$ as a decimal. Show the relation, substitution and units. [2]

2.3 Use denominator 12: $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$.

Exercises 2.4-2.5

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.5 Which description correctly defines prime factor? Choose one answer.

- **A** A prime number that divides the integer exactly.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.4 False. The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.

2.5 Which description correctly defines prime factor? Choose one answer.

- **A** A prime number that divides the integer exactly.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

2.5 A A prime number that divides the integer exactly.

Exercise 3.1

3.1 A supplier has 84 pencils and 126 pens. Find the greatest number of identical bags without leftovers and the contents of each; then explain why the LCM answers a different question. [4]

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3.1 $\text{HCF}=42$, each bag 2 pencils and 3 pens. $\text{LCM}=252$ is a common multiple, useful for matching repeated quantities, not the greatest number of bags dividing both stocks.