Statistics and probability

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · HL

Scope and prerequisites

This handout covers selected HL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Data summaries, histograms and interpretation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** : Frequency divided by class width, used as histogram height.

The method

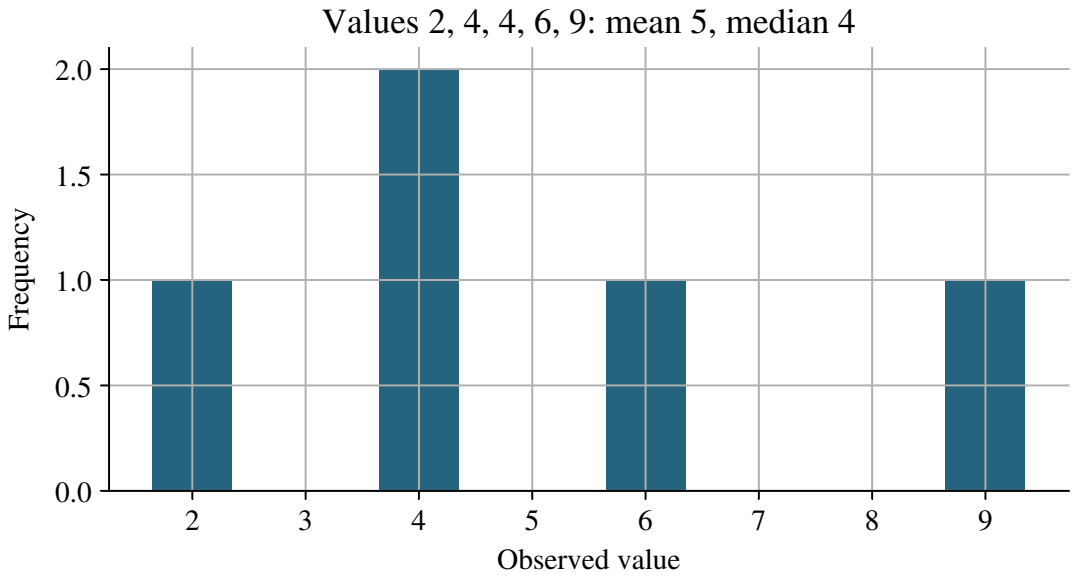
- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

A worked case

$$\overline{x} = \frac{\sum x_i}{n}, \qquad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

Example:

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.



For 2, 4, 4, 6, 9: mean = 25 ÷ 5 = 5, median = 4

■ Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

Not true: A histogram bar's height always equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

■ Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

■ In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Frequency density**: Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

Write from a mean of 7

- A class of width 4 has frequency 32.
- The values 6, 6, 6, 6 and 11 have a mean and a median. Bar height is not always the frequency.

Probability, trees and conditional reasoning

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

The method

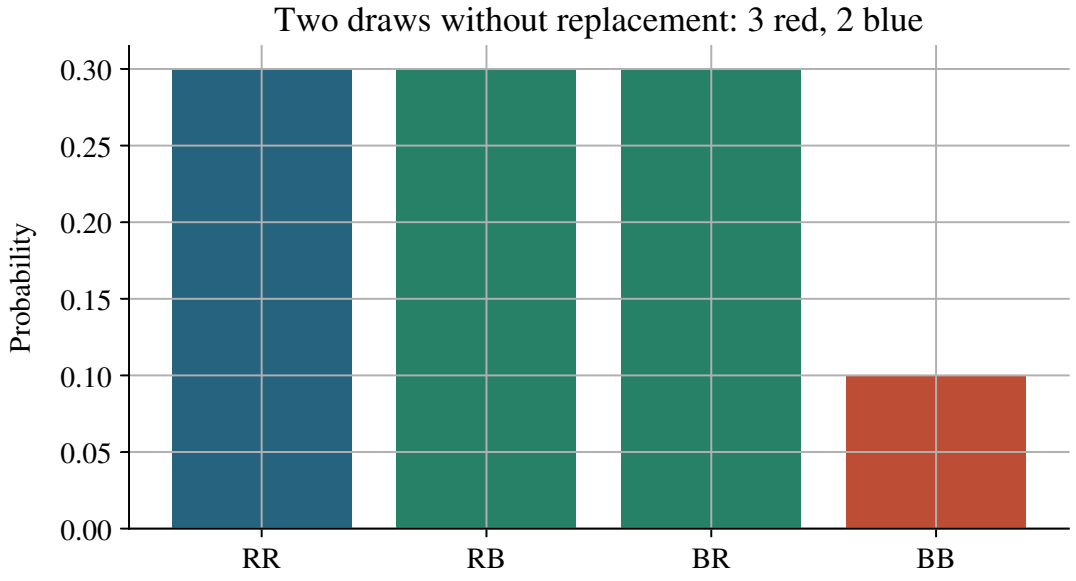
- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

Example:

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.



Without replacement: P(RR) = 3/5 × 2/4 = 0.3 and P(BB) = 2/5 × 1/4 = 0.1

Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so P(A and B)=0≠P(A)P(B).

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product P(A)P(B), justify independence from the context or the supplied information.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

Write from a conditional probability of 0.64

- P(A and B) is 0.16 and P(B) is 0.25.
- Find P(A given B). Mutually exclusive events that can happen are not independent.

Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** : The probability-weighted mean of a random variable.

The method

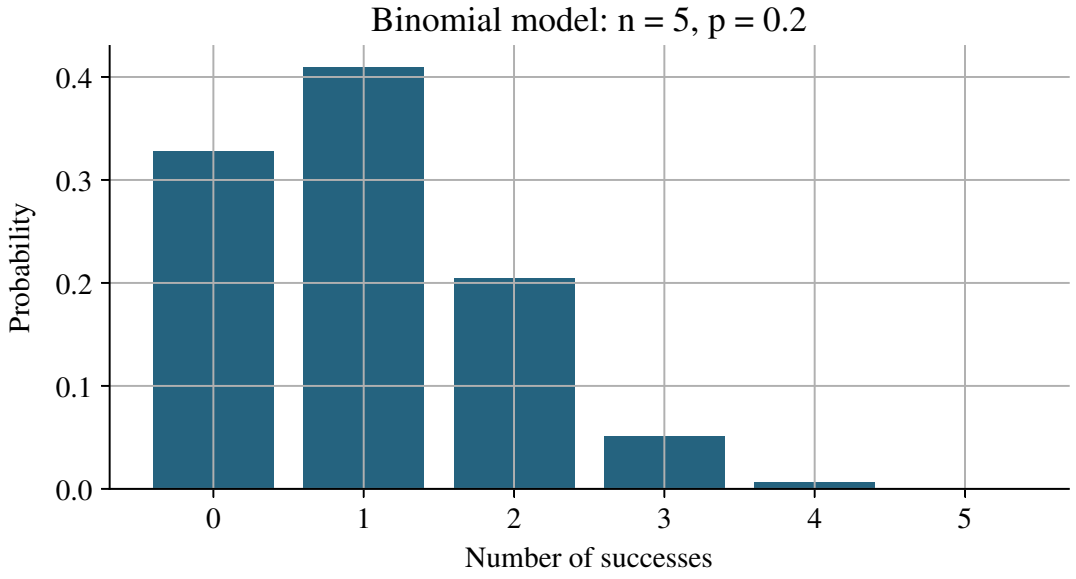
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- E(X)=np and Var(X)=np(1-p).
- For a normal model use z=(x-μ)/σ.
- A Poisson model describes counts with a constant rate and appropriate independence assumptions.

A worked case

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad X \sim B(n, p)$$

Example:

For X binomial(5,0.2), P(X=0)=0.8^5=0.32768, E(X)=1 and Var(X)=0.8. For a normal quantity with μ=100,σ=15, the value 130 has z=2. A Poisson mean of 3 per hour gives mean 6 over two hours.



$X \sim B(5,\ 0.2)$: $P(X = 1) \approx 0.41$ is the largest bar; $P(X = 5)$ is tiny

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Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

Not true: Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X<k)$, $P(X\leq k)$ and a tail complement.
- State assumptions in context.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.

Key:

- **Expected value**: The probability-weighted mean of a random variable.
- A binomial model needs fixed n, independent trials, two outcomes and constant p.

- Watch for: not every count is binomial: changing p or dependence can invalidate it.

Write from a z-score of 3

- A value 82 comes from a distribution with mean 70 and standard deviation 4.
- Find the z-score. A binomial count needs independent trials.

Write from an expected count of 3

- A binomial model has n = 10 trials with p = 0.3.
- The expected count is np = 3 – the probability-weighted mean, not a promise of 3.

Write from a z-score of 2

- A value 60 comes from a distribution with mean 50 and standard deviation 5.
- The z-score is $10 \div 5 = 2$ – two standard deviations above the mean.