

Functions

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · HL

Scope and prerequisites

This handout covers selected HL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Domains, inverses and composition

■ Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

■ The method

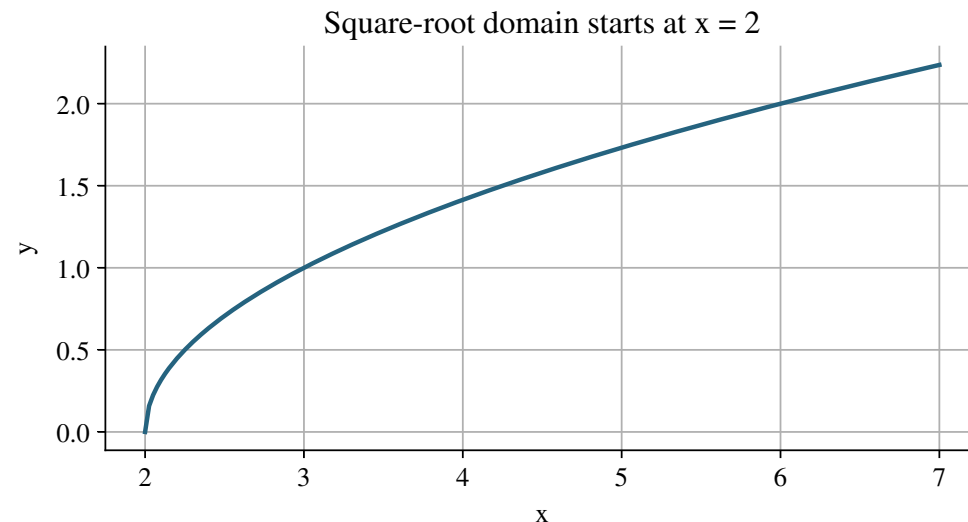
- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .

■ A worked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

Example:

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus f inverse(x)= x^2+2 with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



$y = \sqrt{x - 2}$ exists only for $x \geq 2$, so its domain starts at 2 and its range starts at 0

■ Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

Not true: Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one ($f(1)=f(-1)$ for x^2), so its domain must be restricted first.

■ Applying it

- Check $f(f \text{ inverse}(x))=x$ on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

■ In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Domain:** The set of allowed inputs to a function.

- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

■ Write from an inverse value of 6

- The function $6x$ plus 1 sends 6 to 37.
- A quadratic on all real numbers needs a restricted domain before it has an inverse.

Exponentials, logarithms and modelling

■ Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.

■ The method

- For $y = Ae^{kt}$, k is a proportional rate.
- Taking logs gives $\ln y = \ln A + kt$.
- Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.

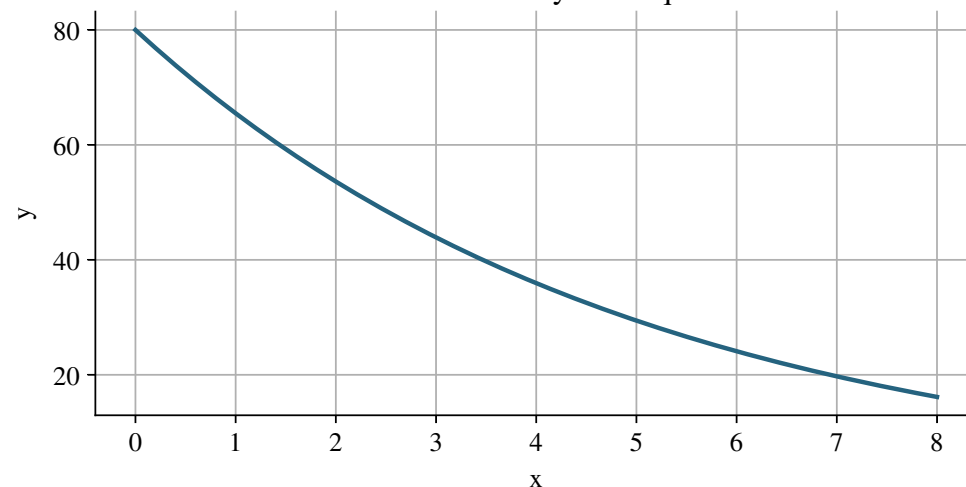
■ A worked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

Example:

If $y = 80e^{-0.2t}$, $y = 40$ gives $e^{-0.2t} = 1/2$. Hence $t = \ln 2 / 0.2 \approx 3.466$. For $3^x = 20$, $x = \ln 20 / \ln 3$. Plotting $\ln y$ against t linearises this exponential model.

Same fractional decay over equal times



$y = 80e^{-0.2x}$ loses the same fraction, about 18%, in every unit of x

■ Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

Not true: For positive x and y , $\ln(x+y) \neq \ln x + \ln y$. The log of a sum has no simple law; the product rule is $\ln(xy) = \ln x + \ln y$.

■ Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

■ In your course

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- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Half-life:** The time for a decaying quantity to fall to half its initial value.
- For $y=Ae^{kt}$, k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

■ **Write from 2 to the power 8**

- Solve 2 to the power $x = 256$.
- The sum of $\ln 2$ and $\ln 9$ is $\ln 18$, not $\ln$ of the sum.

■ **Write from the solution 3**

- Solve 5 to the power $x = 125$.
- Both sides as powers of 5: $x = 3$ – matching bases beat log rules here.

■ **Write from a half-life of 10 days**

- 200 g of a substance decays to 50 g in 20 days.
- Two halvings in 20 days, so the half-life is 10 days – count the halvings, not the subtraction.