

# Functions

International Baccalaureate · IB Diploma · Mathematics: Analysis  
and Approaches · HL

## Scope and prerequisites

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This handout covers selected HL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

## Domains, inverses and composition

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### ■ Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
  - This lesson studies **domain** : The set of allowed inputs to a function.
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### ■ The method

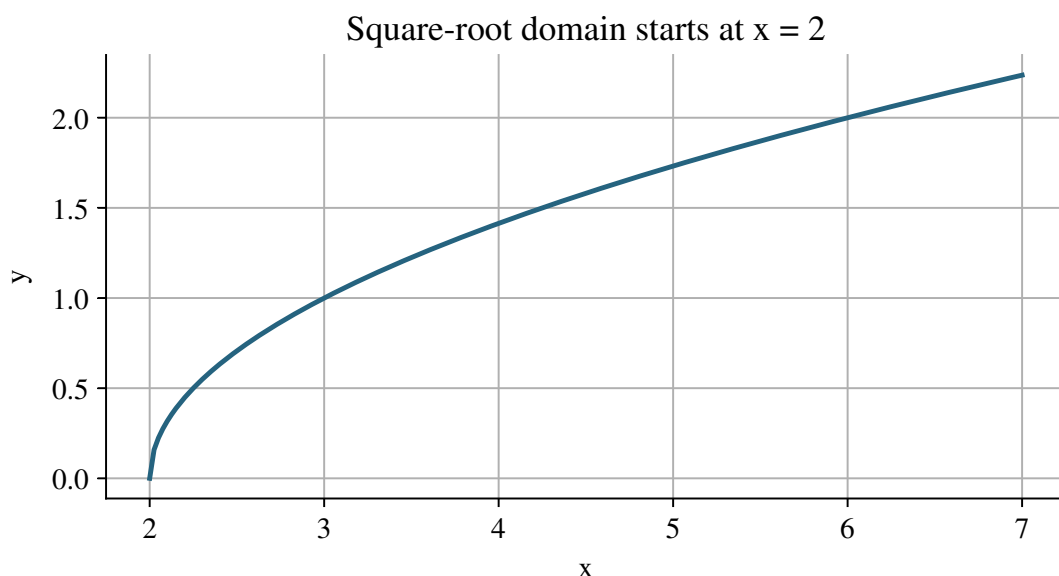
- State the domain and range.
  - For an inverse, first ensure the function is one-to-one on its domain.
  - Composition  $fg$  means apply  $g$  first, then  $f$ ; the intermediate output must be an allowed input to  $f$ .
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### ■ A worked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

#### Example:

For  $f(x)=\sqrt{x-2}$ ,  $x \geq 2$  and the range is  $y \geq 0$ . From  $y=\sqrt{x-2}$ ,  $x=y^2+2$ . Thus  $f$  inverse( $x$ )= $x^2+2$  with  $x \geq 0$ . For  $g(x)=x+3$ ,  $fg(1)=f(4)=\sqrt{2}$ .



$y = \sqrt{x - 2}$  exists only for  $x \geq 2$ , so its domain starts at 2 and its range starts at 0

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### ■ Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside  $f$  has the opposite sign to the graph's movement.

**Not true:** Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one ( $f(1)=f(-1)$  for  $x^2$ ), so its domain must be restricted first.

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### ■ Applying it

- Check  $f(f^{-1}(x))=x$  on the inverse domain.
  - Use a sketch to test whether a horizontal line meets the original graph more than once.
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### ■ In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and  $\pi$  exact and show each step. On calculator papers, still write the expression before its value.

### Key:

- **Domain:** The set of allowed inputs to a function.

- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

### ■ Write from an inverse value of 6

- The function  $6x$  plus 1 sends 6 to 37.
- A quadratic on all real numbers needs a restricted domain before it has an inverse.

## Exponentials, logarithms and modelling

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### ■ Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
  - This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.
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### ■ The method

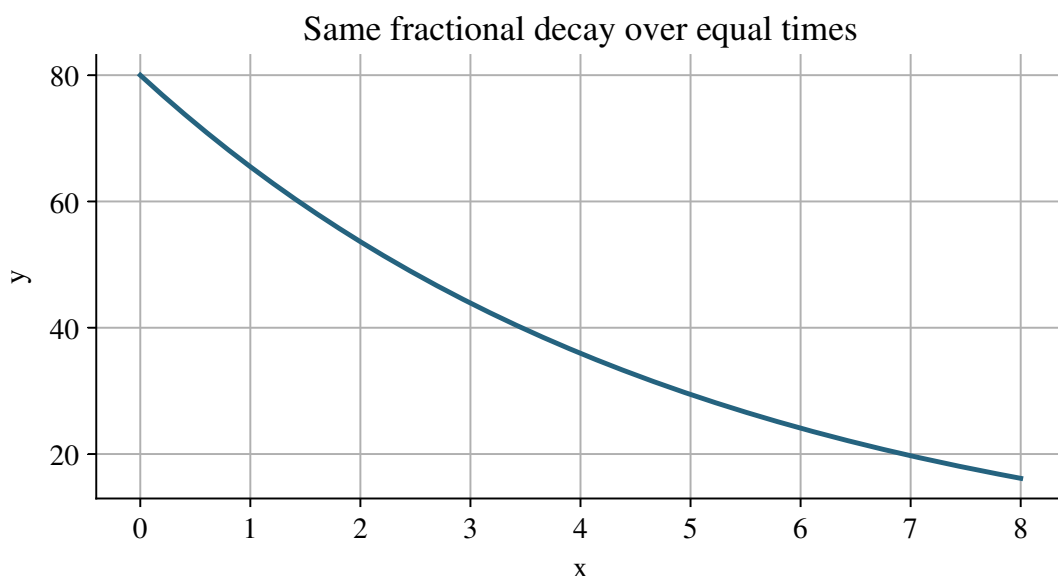
- For  $y = Ae^{kt}$ ,  $k$  is a proportional rate.
  - Taking logs gives  $\ln y = \ln A + kt$ .
  - Logarithms require positive arguments, and  $\log(x+y)$  is not  $\log x + \log y$ .
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### ■ A worked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

#### Example:

If  $y = 80e^{(-0.2t)}$ ,  $y = 40$  gives  $e^{(-0.2t)} = 1/2$ . Hence  $t = \ln 2 / 0.2 \approx 3.466$ . For  $3^x = 20$ ,  $x = \ln 20 / \ln 3$ . Plotting  $\ln y$  against  $t$  linearises this exponential model.



$y = 80e^{(-0.2x)}$  loses the same fraction, about 18%, in every unit of  $x$

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### ■ Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative  $k$  describes decay; a negative starting amount usually contradicts the context.

**Not true:** For positive  $x$  and  $y$ ,  $\ln(x+y) \neq \ln x + \ln y$ . The log of a sum has no simple law; the product rule is  $\ln(xy) = \ln x + \ln y$ .

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### ■ Applying it

- Compare actual observations with the model.
  - Systematic departures may indicate changing conditions.
  - In a report, explain what the rate and initial value mean, rather than giving a bare equation.
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### ■ In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and  $\pi$  exact and show each step. On calculator papers, still write the expression before its value.

**Key:**

- **Half-life:** The time for a decaying quantity to fall to half its initial value.
- For  $y = Ae^{kt}$ ,  $k$  is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

■ **Write from 2 to the power 8**

- Solve 2 to the power  $x = 256$ .
- The sum of  $\ln 2$  and  $\ln 9$  is  $\ln 18$ , not  $\ln$  of the sum.

■ **Write from the solution 3**

- Solve 5 to the power  $x = 125$ .
- Both sides as powers of 5:  $x = 3$  – matching bases beat log rules here.

■ **Write from a half-life of 10 days**

- 200 g of a substance decays to 50 g in 20 days.
- Two halvings in 20 days, so the half-life is 10 days – count the halvings, not the subtraction.