5.7 Maclaurin series

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS Maclaurin series • approximation • term • percentage error • converge • factorial

Objective

Build the skills to answer exam questions on **Maclaurin series** : building series from derivatives, combining known series, and using them for limits and approximations.

You must be able to:

- find a Maclaurin series from $f(0)$, $f'(0)$, $f''(0)$, …
- use the standard series for e^x, $\sin x$, $\cos x$, $\ln(1+x)$ and $(1+x)^p$
- multiply, substitute into and differentiate known series
- use series to estimate values, integrals and limits

1 Key knowledge and worked examples

Maclaurin series

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots$$

function	series	valid for
e^x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	all x
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$	all x
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$	all x
$\ln(1+x)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$	$-1 < x \leq 1$
$(1+x)^p$	$1 + px + \frac{p(p-1)}{2!}x^2 + \cdots$	$\lvert x\rvert < 1$

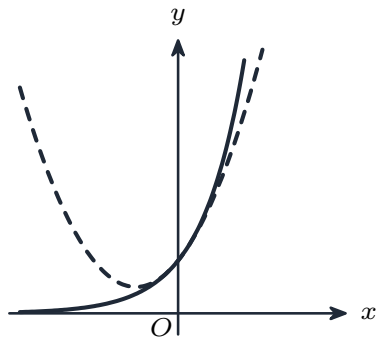
Worked example. e^{x^2}: put x^2 in place of x: $\mathbf{1 + x^2 + \frac{x^4}{2} + \cdots}$.

Answer shapes: when you multiply two series, keep only the powers you were asked for.

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2 Practice

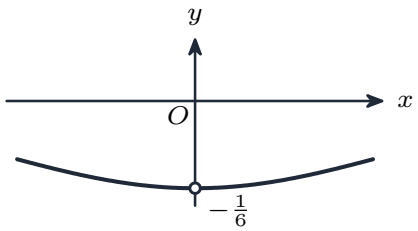
2.1 Write down the first three terms of the Maclaurin series for e^{2x}. [2]



2.2 Use derivatives to find the Maclaurin series for $\ln(1+x)$ up to the term in x^3. [3]

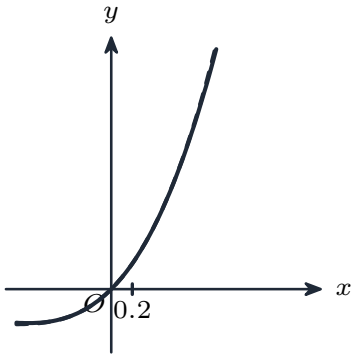
f	f'	f''	$f^{(3)}$
0	?	?	?

2.3 Use series to find $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$. [3]



3 Exam-style questions

3.1 $f(x) = e^x \sin x$.

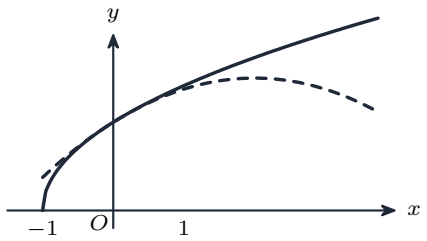


(a) Show that the Maclaurin series for $f(x)$ up to the term in x^3 is $x + x^2 + \frac{1}{3}x^3$. [4]

(b) Use your series to estimate $\displaystyle\int_0^{0.2} e^x \sin x \, dx$. [2]

(c) Explain why the estimate in (b) is good. [2]

3.2 $f(x) = (1 + x)^{1/2}$.



(a) Find the Maclaurin series for $f(x)$ up to the term in x^2. [2]

(b) Hence estimate $\sqrt{1.1}$. [2]

(c) Find the percentage error of your estimate. [2]

(d) Explain why the series cannot be used to estimate $\sqrt{3}$. [2]

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