3.6 Vectors: the vector product and planes

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS vector product • normal vector • plane • Cartesian equation • line of intersection • parallelogram

Objective

Build the skills to answer exam questions on **the vector product and planes** : areas, equations of planes, and intersections of lines and planes.

You must be able to:

- find the vector product $\mathbf{a} \times \mathbf{b}$ and use $|\mathbf{a} \times \mathbf{b}|$ for areas
- write the equation of a plane in vector and Cartesian form
- find the angle between a line and a plane, and between two planes
- find the intersection of a line and a plane, and of two or three planes

1 Key knowledge and worked examples

■ Vector product and planes

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}, \qquad |\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$$

$\mathbf{a} \times \mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. Area of a parallelogram $= |\mathbf{a} \times \mathbf{b}|$; of a triangle $= \frac{1}{2}|\mathbf{a} \times \mathbf{b}|$.

object	equation
plane with normal $\mathbf{n}$ through A	$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$, or $ax + by + cz = d$
plane through A, B, C	$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC}$
angle between line (direction $\mathbf{b}$) and plane	$\sin\theta = \dfrac{\lvert\mathbf{b} \cdot \mathbf{n}\rvert}{\lvert\mathbf{b}\rvert\lvert\mathbf{n}\rvert}$
line of two planes	direction $\mathbf{n}_1 \times \mathbf{n}_2$; find one point by setting $z = 0$

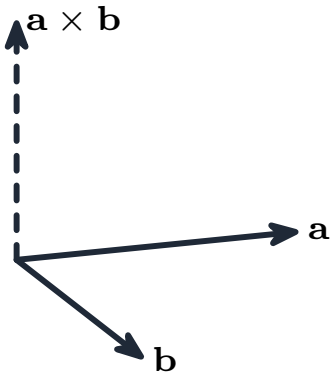
Line meets plane: substitute the line's x, y, z (in terms of λ) into the plane's equation.

Worked example. $\begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}0\\1\\4\end{pmatrix} = \begin{pmatrix}8-3\\0-4\\1-0\end{pmatrix} = \begin{pmatrix}5\\-4\\1\end{pmatrix}$.

Answer shapes: check a normal by dotting it with both direction vectors: each gives 0.

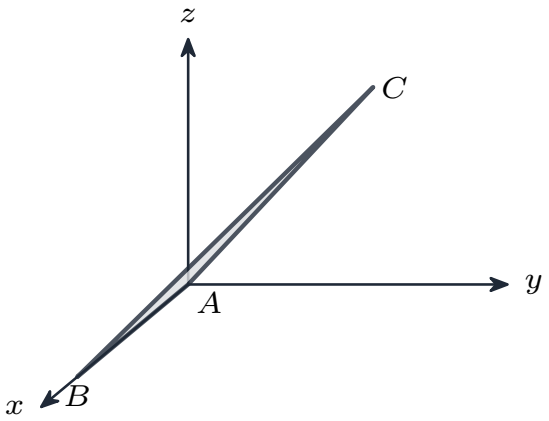
2 Practice

2.1 Find $\begin{pmatrix}2\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\3\\-1\end{pmatrix}$. [2]

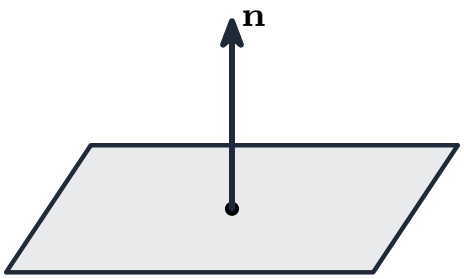


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2.2 Find the area of the triangle with vertices $A(0,0,0)$, $B(2,0,0)$ and $C(0,3,4)$. [3]



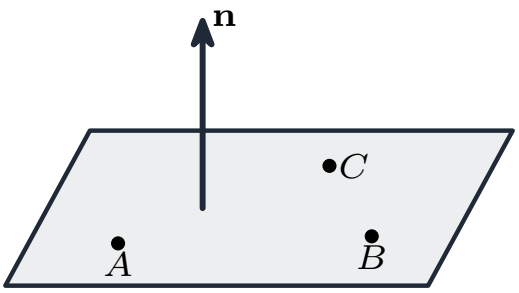
2.3 Find the Cartesian equation of the plane through $(1,2,-1)$ with normal $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$. [3]



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3 Exam-style questions

3.1 The points $A(1, 0, 2)$, $B(3, 1, 0)$ and $C(2, -1, 1)$ lie in a plane Π.



(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

(b) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. [2]

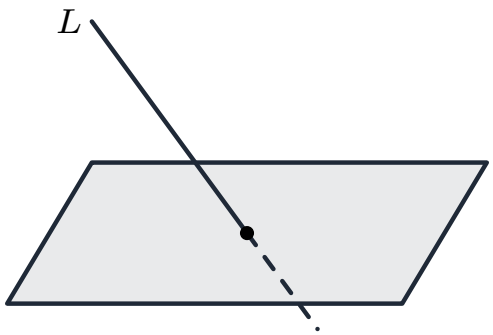
(c) Find the Cartesian equation of Π. [2]

(d) Find the area of triangle ABC. [2]

(e) Find the point where the line $\mathbf{r} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ meets Π. [2]

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3.2 The line L is $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and the plane Π_1 is $x + 2y - z = 7$.



(a) Find the acute angle between L and Π_1. [4]

(b) Find the point where L meets Π_1. [3]

(c) The plane Π_2 is $x - y + z = 2$. Find a vector equation of the line of intersection of Π_1 and Π_2. [3]

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