2.3 Transformations of graphs and rational functions

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS translation • stretch • reflection • rational function • vertical asymptote • image

Objective

Build the skills to answer exam questions on **transformations of graphs and rational functions** .

You must be able to:

- apply translations, stretches and reflections to a graph, and describe them in words
- find the image of a point under one or more transformations
- find the asymptotes and intercepts of $y = \dfrac{ax+b}{cx+d}$ and sketch it

1 Key knowledge and worked examples

■ Transformations and rational functions

new graph	**transformation of $y = f(x)$**
$y = f(x) + b$	translation b up
$y = f(x - a)$	translation a to the right
$y = pf(x)$	vertical stretch, scale factor p
$y = f(qx)$	horizontal stretch, scale factor $\frac{1}{q}$
$y = -f(x)$ / $y = f(-x)$	reflection in the x-axis / y-axis

Changes **inside** the bracket act on x and work "the opposite way".

Rational function $y = \dfrac{ax+b}{cx+d}$: vertical asymptote $x = -\dfrac{d}{c}$ (bottom $= 0$), horizontal asymptote $y = \dfrac{a}{c}$, x-intercept where the top $= 0$, y-intercept $\dfrac{b}{d}$.

Worked example 1. The point $(4, 5)$ on $y = f(x)$ moves under $y = 2f(x-1) + 3$ to $(4+1,\ 2\times 5+3) = (\mathbf{5}, \mathbf{13})$.

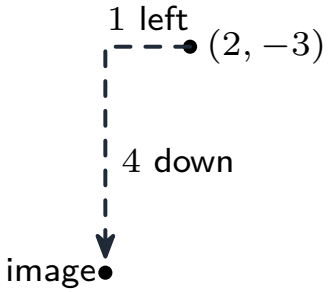
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Worked example 2. $y = \dfrac{2x+1}{x-3}$: asymptotes $x = \mathbf{3}$ and $y = \mathbf{2}$; intercepts $\left(-\frac{1}{2}, 0\right)$ and $\left(0, -\frac{1}{3}\right)$.

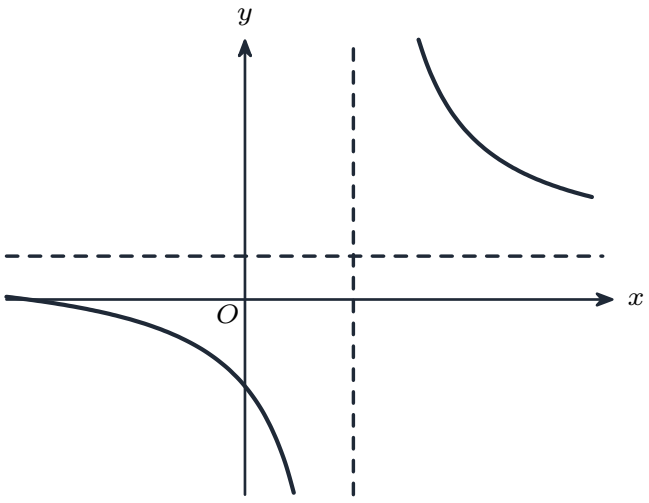
Answer shapes: name each transformation with its size and direction ("vertical stretch, scale factor 3").

2 Practice

2.1 The point $(2, -3)$ lies on $y = f(x)$. Find its image on the graph of $y = f(x+1)-4$. [2]

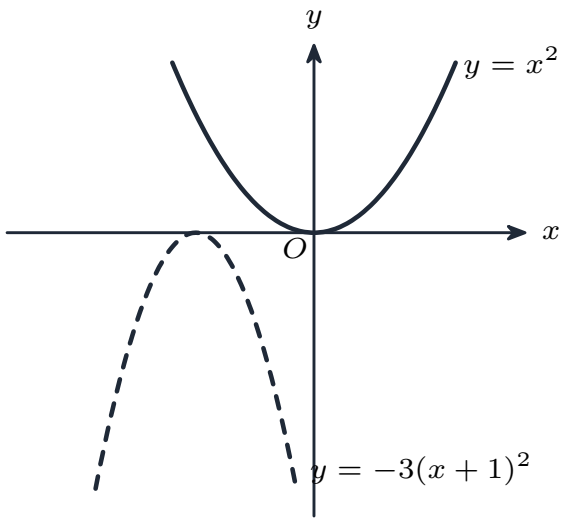


2.2 For $y = \dfrac{x+4}{x-2}$, write down the equations of the asymptotes, and find the intercepts with the axes. [3]



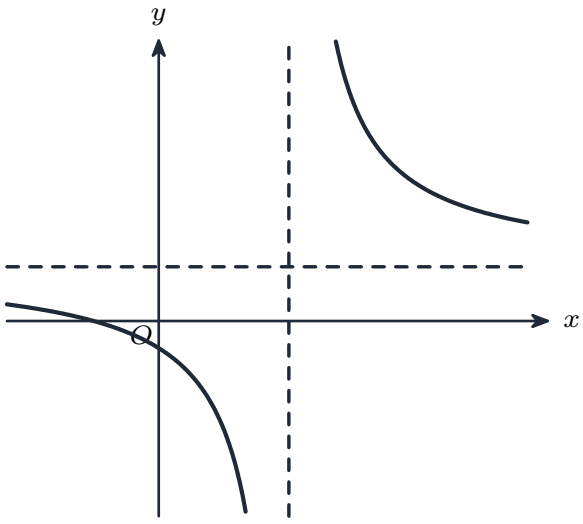
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2.3 Describe a sequence of transformations that maps the graph of $y = x^2$ onto the graph of $y = -3(x+1)^2$. [3]



3 Exam-style questions

3.1 $f(x) = \dfrac{3}{x-2} + 1,\ x \neq 2.$



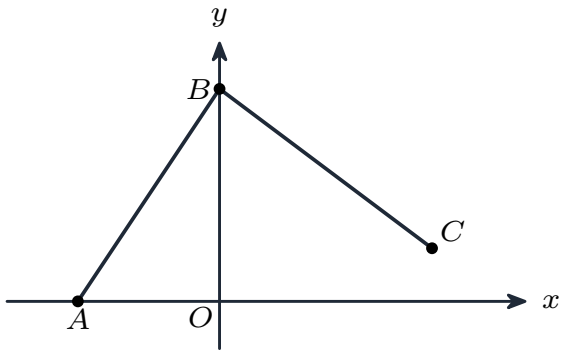
(a) Write down the equations of the asymptotes. [2]

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(b) Find the coordinates of the intercepts with the axes. [2]

(c) Describe the transformations that map $y = \frac{1}{x}$ onto $y = f(x)$. [3]

3.2 The graph of $y = f(x)$, $-2 \leq x \leq 3$, consists of two straight line segments joining $A(-2, 0)$, $B(0, 4)$ and $C(3, 1)$.



(a) Write down the image of B on the graph of $y = f(x) - 3$. [1]

(b) Write down the image of C on the graph of $y = f(2x)$. [1]

(c) Find the images of A, B and C on the graph of $y = -f(x + 1)$. [3]

(d) Write down the range of $y = -f(x + 1)$. [1]

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(e) Find the range of $y = 2f(x) + 1$. [2]

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