# 5.7 Maclaurin series

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

**KEY WORDS** Maclaurin series • approximation • term • percentage error • converge • factorial

## Objective

Build the skills to answer exam questions on **Maclaurin series** : building series from derivatives, combining known series, and using them for limits and approximations.

**You must be able to:**

- find a Maclaurin series from $f(0)$, $f'(0)$, $f''(0)$, …
- use the standard series for $e^x$, $\sin x$, $\cos x$, $\ln(1+x)$ and $(1+x)^p$
- multiply, substitute into and differentiate known series
- use series to estimate values, integrals and limits

## 1 Key knowledge and worked examples

### ■ Maclaurin series

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots$$

| function | series | valid for |
|---|---|---|
| $e^x$ | $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ | all $x$ |
| $\sin x$ | $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ | all $x$ |
| $\cos x$ | $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ | all $x$ |
| $\ln(1+x)$ | $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$ | $-1 < x \leq 1$ |
| $(1+x)^p$ | $1 + px + \frac{p(p-1)}{2!}x^2 + \cdots$ | $\lvert x\rvert < 1$ |

**Worked example.** $e^{x^2}$: put $x^2$ in place of $x$: $\mathbf{1 + x^2 + \frac{x^4}{2} + \cdots}$.

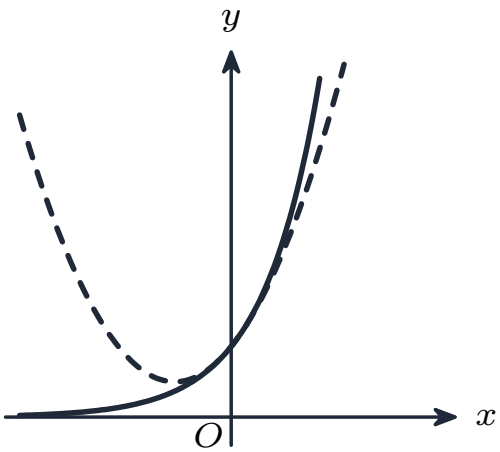
**Answer shapes:** when you multiply two series, keep only the powers you were asked for.



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## 2 Practice

**2.1** Write down the first three terms of the Maclaurin series for $e^{2x}$. [2]

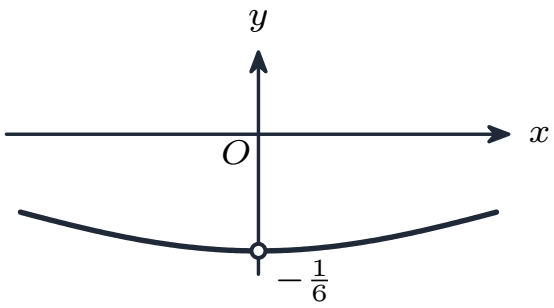



**2.2** Use derivatives to find the Maclaurin series for $\ln(1+x)$ up to the term in $x^3$. [3]

| $f$ | $f'$ | $f''$ | $f^{(3)}$ |
|---|---|---|---|
| 0 | ? | ? | ? |

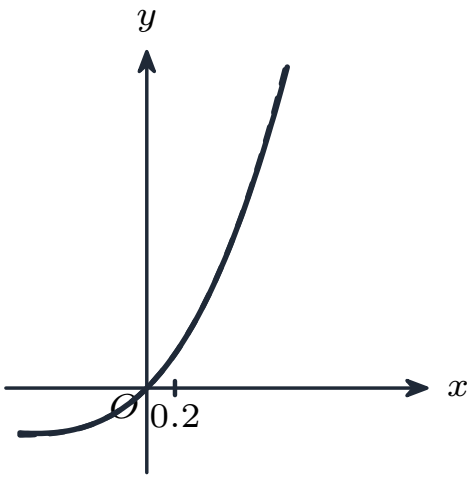

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**2.3** Use series to find $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$. [3]




## 3 Exam-style questions

**3.1** $f(x) = e^x \sin x$.




(a) Show that the Maclaurin series for $f(x)$ up to the term in $x^3$ is $x + x^2 + \frac{1}{3}x^3$. [4]

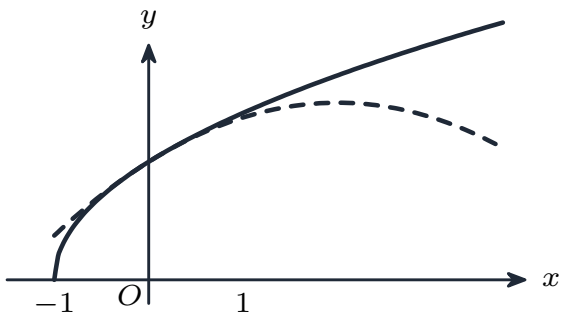


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(b) Use your series to estimate $\int_0^{0.2} e^x \sin x \, dx$. [2]

(c) Explain why the estimate in (b) is good. [2]

**3.2** $f(x) = (1 + x)^{1/2}$.




(a) Find the Maclaurin series for $f(x)$ up to the term in $x^2$. [2]

(b) Hence estimate $\sqrt{1.1}$. [2]

(c) Find the percentage error of your estimate. [2]

(d) Explain why the series cannot be used to estimate $\sqrt{3}$. [2]


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## Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

**2.1**

replace $x$ by $2x$; $\mathbf{1 + 2x + 2x^2}$

**2.2**

$f(0) = 0$, $f'(0) = 1$, $f''(0) = -1$, $f^{(3)}(0) = 2$; $\mathbf{x - \frac{x^2}{2} + \frac{x^3}{3}}$

**2.3**

$\sin x - x = -\frac{x^3}{6} + \frac{x^5}{120} - \cdots$; $\dfrac{\sin x - x}{x^3} = -\frac{1}{6} + \frac{x^2}{120} - \cdots$; limit $\mathbf{-\frac{1}{6}}$

**3.1**

(a) $\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(x - \frac{x^3}{6}\right)$; $x + x^2 + \frac{x^3}{2} - \frac{x^3}{6} = x + x^2 + \frac{1}{3}x^3$

(b) $\left[\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{12}\right]_0^{0.2} = \mathbf{0.0228}$

(c) $x$ is small (at most 0.2) (R1), so the missing terms in $x^4$ and above are very small (R1)

**3.2**

(a) $1 + \frac{1}{2}x + \dfrac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}x^2 = \mathbf{1 + \frac{x}{2} - \frac{x^2}{8}}$

(b) $x = 0.1$; $1 + 0.05 - 0.00125 = \mathbf{1.04875}$

(c) $\dfrac{|1.04875 - 1.048809|}{1.048809} \times 100 = \mathbf{0.0056}\%$

(d) $\sqrt{3}$ needs $x = 2$; the series is valid only for $|x| < 1$ (R1)



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