5.2 Integrals, area and accumulation

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS antiderivative

Objective

Integrals, area and accumulation. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** : A function whose derivative equals the given integrand.

The method

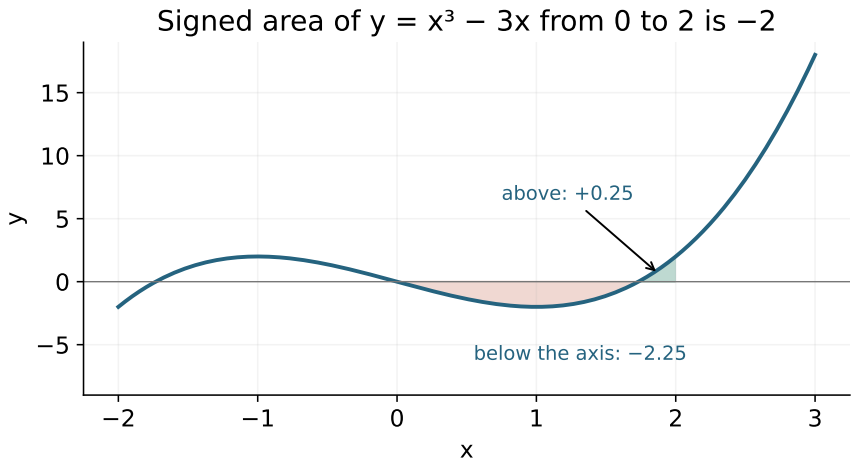
- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

A worked case

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

Example:

For v(t)=3t²-3 over 0≤t≤2, displacement=[t³-3t]_0^2=2. Since v changes sign at t=1, distance=-[t³-3t]_0^1+[t³-3t]_1^2=2+4=6. The constants cancel only for a definite integral.



Where it goes wrong

- The integral of 1/x is ln|x|+C, not the power rule with n=-1.
- Signed area can be zero even when the total enclosed area is positive.

Not true: A definite integral always equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Antiderivative**: A function whose derivative equals the given integrand.
- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- Watch for: the integral of 1/x is ln|x|+C, not the power rule with n=-1.

■ **Write from an integral of 9**

- Find the integral of 2x from 0 to 3.
- For v = t minus 2, from t = 0 to 4, find the displacement and the distance. A signed integral is not the total distance.

2 Practice

2.1 Find the integral of $3x^2$ from 0 to 2. Show the relation, substitution and units. [2]

2.2 For $v=3t^2-3$, find displacement from t=0 to 2. Show the relation, substitution and units. [2]

2.3 For that motion, find total distance from t=0 to 2. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Antiderivative means a function whose derivative equals the given integrand. [2]

2.5 For $v(t)=3t^2-3$ on $0\le t\le 2$, why are displacement and total distance different? Choose one answer.

- **A** Distance equals displacement whenever the velocity is a polynomial
- **B** Displacement ignores time, so it cannot be found by integrating v(t)
- **C** The velocity changes sign at t=1, so the signed integral cancels part of the travel [1]

3 Exam-style questions

3.1 For velocity $v=t-2$ on $0\le t\le 5$, find displacement and total distance and explain the distinction. [4]

Solutions

2.1 An antiderivative is x^3. Subtract its values: 2^3-0^3=8.

2.2 Displacement=$[t^3$-3t] from 0 to 2=8-6=2.

2.3 Velocity changes sign at t=1. The two positive distances are 2 and 4, giving 6.

2.4 True. True: A function whose derivative equals the given integrand.

2.5 C The signed integral over the whole interval gives displacement 2. The motion reverses at t=1, so total distance needs the split: 2 backwards then 4 forwards, totalling 6.

3.1 Integral$[t^2/2-2\ t]_0{}^5$=2.5. Sign changes at 2; signed integrals are −2 then 4.5, so distance 6.5. Units follow velocity/time units. Distance integrates |v|.

exercises.lol exercises.com.cn