5.1 Derivatives and stationary points

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS derivative

Objective

Derivatives and stationary points. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.

The method

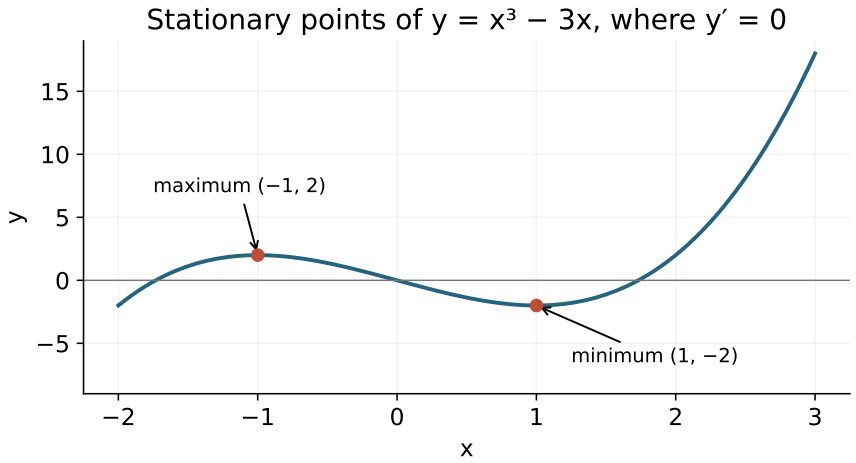
- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad f'(x) = 0$$

Example:

For $y=x^3-3x$, $dy/dx=3x^2-3$. At x=1, the gradient is 0 and y=-2. The second derivative is 6x, positive at x=1, so this is a local minimum. At x=-1, y=2 and the second derivative is negative, giving a local maximum.



Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

Not true: Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at x=0 is neither a maximum nor a minimum.

Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

■ **Write from a derivative of 8**

- For y = x squared minus 10x, find the derivative at x = 9.
- Find the stationary value. A derivative of 0 is not always a turning point.

2 Practice

2.1 For $y=x^3-3x$, find dy/dx at x=2. Show the relation, substitution and units. [2]

2.2 Find the y-coordinate of the stationary point at x=1. Show the relation, substitution and units. [2]

2.3 For $y=4x^3$, find dy/dx at x=2. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Derivative means the instantaneous rate of change, also the gradient of a tangent. [2]

2.5 At x=0 the curve $y=x^3$ has dy/dx=0. Why is it not a local maximum or minimum? Choose one answer.

- **A** The gradient does not change sign: the curve keeps increasing through the point
- **B** Every stationary point is a maximum or minimum; x=0 is a minimum of x^3
- **C** The derivative $3x^2$ is undefined at x=0, so the point is not stationary [1]

3 Exam-style questions

3.1 For $y=x^3-3x$ on $[-2, 2]$, find the absolute maximum and minimum, including all candidates. [4]

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Solutions

2.1 dy/dx=3x^2-3. At x=2 it is 12-3=9.

2.2 Substitute into the original curve: y(1)=1-3=-2.

2.3 The derivative is 12x^2; at 2 it is 12×4=48.

2.4 True. True: The instantaneous rate of change, also the gradient of a tangent.

2.5 A The derivative 3x^2 is positive on both sides of x=0, so the point is stationary but the function continues to rise - a point of inflection. Classification needs the sign change or the second derivative, not merely dy/dx=0.

3.1 Derivative 3 x^2−3=0 at±1. Values y(−2)=−2, y(−1)=2, y(1)=−2, y(2)=2. Maximum 2 at −1, 2; minimum−2 at −2, 1. Checking only stationary points misses endpoint ties.

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