4.4 The normal distribution

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS normal distribution • mean • standard deviation • inverse normal • standardize • symmetric

Objective

Build the skills to answer exam questions on **the normal distribution** : probabilities, inverse normal calculations and standardized z-values.

You must be able to:

- describe the shape of a normal distribution and use 68%, 95% and 99.7%
- find normal probabilities with the GDC
- use the inverse normal to find a value from a probability
- use $z = \dfrac{x - \mu}{\sigma}$ to find an unknown mean or standard deviation

1 Key knowledge and worked examples

■ The normal distribution

$X \sim N(\mu, \sigma^2)$ is bell-shaped and symmetric about the mean μ. About 68% of values are within 1σ of the mean, 95% within 2σ and 99.7% within 3σ.

you know	you want	GDC
a value x	a probability	normal cdf (lower, upper, μ, σ)
a probability	a value x	inverse normal (area to the left, μ, σ)
a probability, but μ or σ unknown	μ or σ	inverse normal with $N(0,1)$, then $z = \dfrac{x - \mu}{\sigma}$

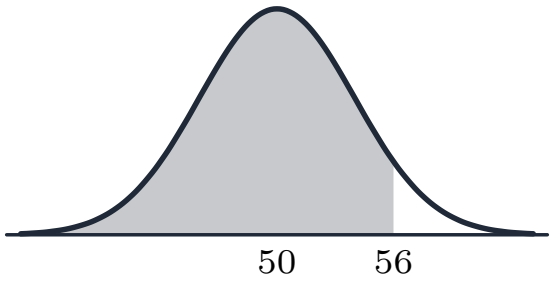
Worked example. $X \sim N(20, 3^2)$: $P(X > 23) = P(Z > 1) = \mathbf{0.159}$.

Answer shapes: sketch the curve and shade the area first; it shows whether the answer should be more or less than 0.5.

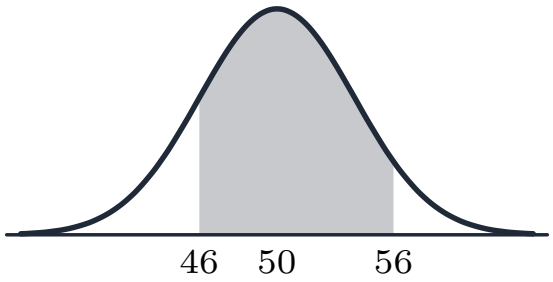
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2 Practice

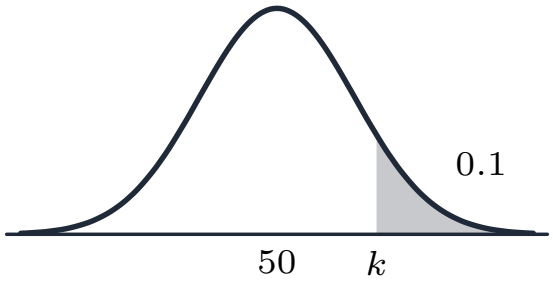
2.1 $X \sim N(50, 4^2)$. Find $P(X < 56)$. [2]



2.2 $X \sim N(50, 4^2)$. Find $P(46 < X < 56)$. [3]



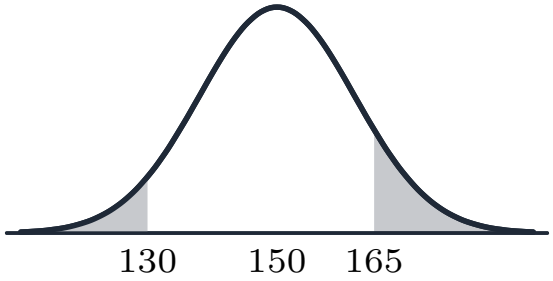
2.3 $X \sim N(50, 4^2)$. Find k such that $P(X > k) = 0.1$. [3]



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3 Exam-style questions

3.1 The masses of apples from a farm are normally distributed with mean 150 g and standard deviation 12 g.



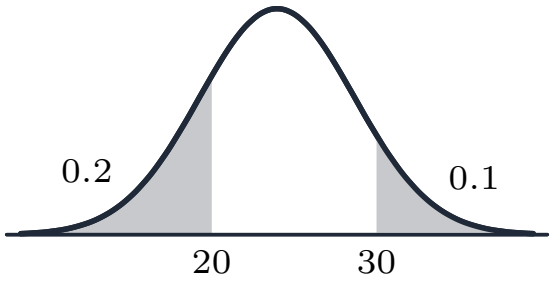
(a) Find the probability that an apple has a mass of more than 165 g. [2]

(b) Apples under 130 g are rejected. Find the expected number rejected in a batch of 400. [3]

(c) The heaviest 5% of apples are sold as "premium". Find the least mass of a premium apple. [2]

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3.2 The times, T minutes, that students take to finish a puzzle are normally distributed. $P(T < 20) = 0.2$ and $P(T > 30) = 0.1$.



(a) Write down two equations in μ and σ. [3]

(b) Find μ and σ. [2]

(c) Find $P(T > 25)$. [2]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

normal cdf with $\mu = 50$, $\sigma = 4$; $\mathbf{0.933}$

2.2

$0.9332 - 0.1587 = \mathbf{0.775}$

2.3

$P(X < k) = 0.9$; inverse normal; $k = \mathbf{55.1}$

3.1

(a) $P(M > 165) = \mathbf{0.106}$

(b) $P(M < 130) = 0.0478$; $400 \times 0.0478 = \mathbf{19.1}$ (accept 19)

(c) $P(M < m) = 0.95$; $m = \mathbf{170}$ g (169.7)

3.2

(a) z-values -0.8416 and 1.2816; $\dfrac{20 - \mu}{\sigma} = -0.8416$ and $\dfrac{30 - \mu}{\sigma} = 1.2816$

(b) $\sigma = \mathbf{4.71}$; $\mu = \mathbf{24.0}$

(c) normal cdf with $\mu = 23.96$, $\sigma = 4.710$; $\mathbf{0.413}$