4.2 Probability, trees and conditional reasoning

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS conditional probability

Objective

Probability, trees and conditional reasoning. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

■ The method

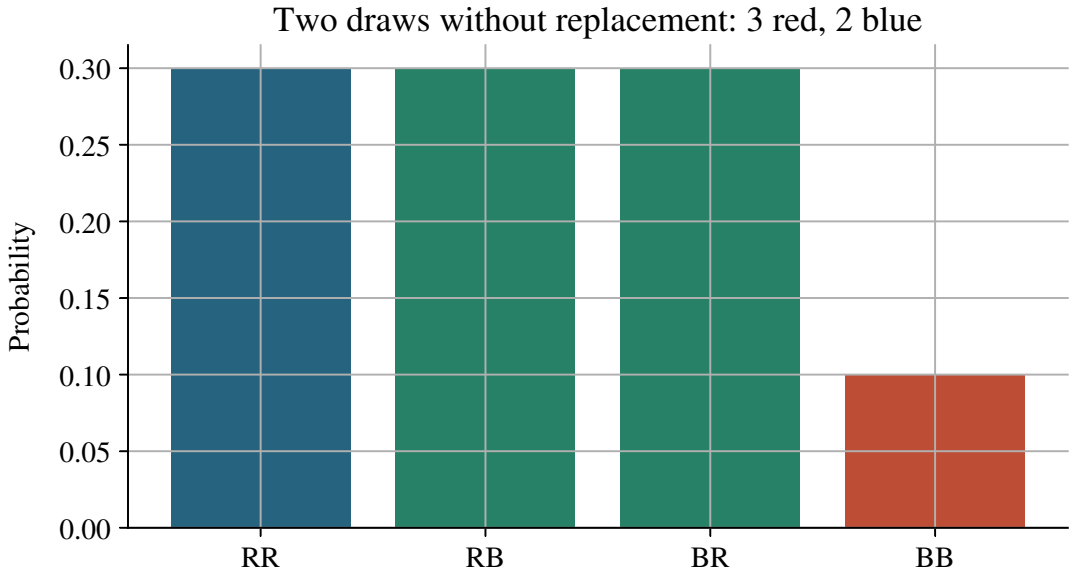
- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

■ A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

Example:

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.



Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

■ **Write from a conditional probability of 0.64**

- P(A and B) is 0.16 and P(B) is 0.25.
- Find P(A given B). Mutually exclusive events that can happen are not independent.

2 Practice

2.1 Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

2.2 Find P(A given B) if P(A∩B)=0.12 and P(B)=0.3. Show the relation, substitution and units. [2]

2.3 Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Conditional probability means the probability of an event after restricting the sample space to a stated condition. [2]

2.5 Two disjoint events A and B both have positive probability. Can they be independent? Choose one answer.

- **A** Yes: independence is automatic for all events
- **B** Yes: disjoint and independent mean the same thing
- **C** No: if A happens, P(B) drops to 0, so knowing A changes B [1]

3 Exam-style questions

3.1 A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

Solutions

2.1 Without replacement, $P(RR)=3/5\times2/4=0.3$.

2.2 $P(A|B)=P(A\cap B)/P(B)=0.12/0.3=0.4$.

2.3 Add the two orders: $3/5\times2/4+2/5\times3/4=0.6$.

2.4 True. True: The probability of an event after restricting the sample space to a stated condition.

2.5 C Disjoint means A and B cannot happen together, so P(B given A)$=0\neq P(B)$. Independence requires that knowing one does not change the other - disjoint events with positive probability always fail that.

3.1 $P(RR)=3/5\times2/4=0.3$. P(at least oneR)$=1-2/5\times1/4=0.9$. Conditional probability$=0.3/0.9=1/3$. The conditioning changes the sample space.