3.4 Vectors and transformation geometry

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS resultant

Objective

Vectors and transformation geometry. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** : The vector sum representing the combined displacement or force.

The method

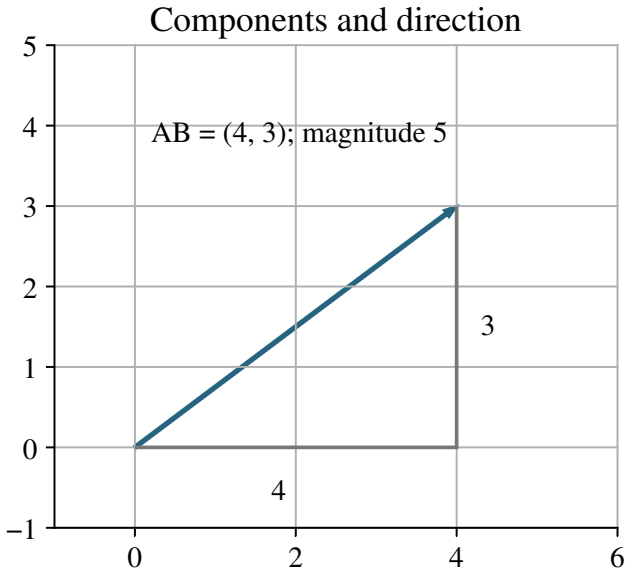
- Add corresponding vector components.
- A displacement from A to B is OB-OA.
- Parallel vectors are scalar multiples.
- A vector has magnitude and direction; a distance is a scalar.

A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \qquad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

Example:

With a=(4,1) and b=(1,3), a+b=(5,4). Their scalar product is a · b=4×1+1×3=7. The magnitude of a is √17. For A=(1,2), B=(5,5), AB=(4,3) and |AB|=5.



Where it goes wrong

- The order of subtraction matters: BA=-AB.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

Not true: AB and BA always have the same components. BA goes the opposite way to AB, so BA=−AB: every component changes sign.

Applying it

- For a translation, every point uses the same displacement.
- For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Resultant**: The vector sum representing the combined displacement or force.
- Add corresponding vector components.
- Watch for: the order of subtraction matters: BA=-AB.

■ **Write from an x-component of 9**

- Add the vectors (3, 1) and (6, 2).
- Find the magnitude of (6, 8) and the scalar product. AB and BA do not have the same components.

2 Practice

2.1 Find the x-component of (4,1)+(1,3). Show the relation, substitution and units. [2]

2.2 Find the magnitude of (4,3). Show the relation, substitution and units. [2]

2.3 Find the scalar product of (4,1) and (1,3). Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Resultant means the vector sum representing the combined displacement or force. [2]

2.5 What is the displacement vector from A=(1,2) to B=(5,5)? Choose one answer.

- **A** (3,4): subtract the start from the end and then swap the components
- **B** (6,7): add the two position vectors to get the displacement
- **C** (4,3): subtract the start position from the end position [1]

3 Exam-style questions

3.1 Points A=(1, 2, 0), B=(3, 2, 1), C=(1, 5, 0). Find AB · AC and decide whether the angle at A is right; distinguish a zero vector exception. [4]

Solutions

2.1 Add the x-components: 4+1=5.

2.2 Magnitude=$\sqrt{(4^2+3^2)}$=5.

2.3 Multiply corresponding components and add: 4×1+1×3=7.

2.4 True. True: The vector sum representing the combined displacement or force.

2.5 C AB = OB - OA = (5,5)-(1,2) = (4,3). The order matters: BA would be (-4,-3), the opposite direction.

3.1 AB=(2, 0, 1), AC=(0, 3, 0), dot 0. Both nonzero, so angle$\pi/2$. A zero dot product involving a zero vector would not define an angle.