3.3 Radians, identities and trigonometric equations

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS radian

Objective

Radians, identities and trigonometric equations. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.

The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

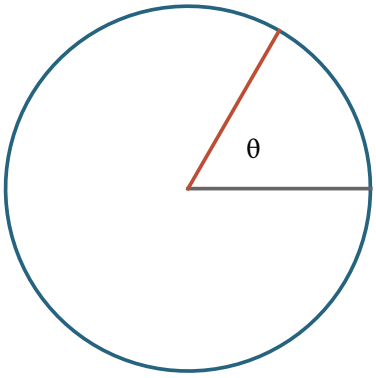
A worked case

$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

Example:

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius × angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

Not true: The principal inverse sine value is always the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Watch for: A calculator's inverse sine gives one principal value.

■ **Write from an arc length of 3**

- A circle has radius 6 and the angle is 0.5 radians.
- Find the arc length and the sector area. The principal inverse sine is not the only solution in a full turn.

2 Practice

2.1 For r=4 and θ=0.5 radians, find arc length. Show the relation, substitution and units. [2]

2.2 For r=4 and θ=0.5 radians, find sector area. Show the relation, substitution and units. [2]

2.3 How many solutions does sin x=0.5 have on $0 \leq x < 2\pi$? Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: The principal inverse sine value is always the only solution in a full turn. [2]

2.5 Solving sin x=1/2 on $0 \leq x < 2\pi$, why are there two solutions? Choose one answer.

- **A** Every equation has exactly two solutions
- **B** Sine is positive in two quadrants, so $\pi/6$ and $5\pi/6$ both work
- **C** The calculator inverse gives both values at once [1]

3 Exam-style questions

3.1 Solve $2\sin^2 x - \sin x - 1 = 0$ for $0 \leq x < 2\pi$, showing why no solutions are lost. [4]

Solutions

2.1 Arc length=rθ=4×0.5=2.

2.2 Sector area=$r^2\theta/2$=16×0.5/2=4.

2.3 The two solutions are $\pi/6$ and $5\pi/6$ in the stated interval.

2.4 False. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

2.5 B The inverse sine returns only the principal value $\pi/6$. Sine is also positive in the second quadrant, giving π-$\pi/6$=$5\pi/6$. Both must be listed within the stated interval.

3.1 Factor(2 sinx+1)(sinx−1)=0. sinx=1 gives$\pi/2$; sinx=−1/2 gives $7\pi/6$, $11\pi/6$. Both allowed roots are considered and the endpoint interval is respected.