3.1 Angle reasoning, similarity and mensuration

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS scale factor

Objective

Angle reasoning, similarity and mensuration. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

The method

- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

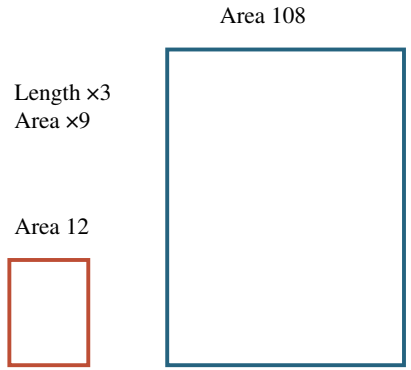
A worked case

$$\frac{A_2}{A_1} = k^2, \qquad \frac{V_2}{V_1} = k^3$$

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12\times3^2=108$ cm^2 and a model volume of 8 cm^3 gives $8\times3^3=216$ cm^3. A cylinder with r=3,h=5 has volume $\pi r^2h=45\pi$.

A length scale factor is squared for area



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

Not true: Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Scale factor**: The multiplier that relates corresponding lengths in similar shapes.
- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- Watch for: equal angles alone establish similarity, not equal size.

■ **Write from a volume factor of 125**

- A length scale factor is 5.
- Find the area factor and the volume factor. Doubling lengths does not double the volume.

2 Practice

2.1 Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

2.2 Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

2.3 Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Scale factor means the multiplier that relates corresponding lengths in similar shapes. [2]

2.5 Two similar solids have length scale factor 3. Why is the volume factor 27? Choose one answer.

- **A** Volume scales with the cube of the length factor: $3^3=27$
- **B** Volume scales with the length factor itself: a factor of 3 gives 3
- **C** Volume scales with the square of the length factor: $3^2=9$, then add the base [1]

3 Exam-style questions

3.1 Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

Solutions

2.1 Area scales by the square of the length factor: $3^2=9$.

2.2 Volume scales by the cube: $3^3=27$.

2.3 Interior-angle sum$=(5\text{-}2)\times180=540°$.

2.4 True. True: The multiplier that relates corresponding lengths in similar shapes.

2.5 A Each of the three dimensions multiplies by 3, so volume multiplies by $3\times3\times3=27$. Area uses the square and volume the cube of the length factor.

3.1 Length ratio 3:2, area 9:4. Area scales as the square and volume as the cube of the length factor, assuming genuine geometric similarity.