2.5 Modulus, inequalities and symmetry of functions

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS modulus • inequality • even function • odd function • reciprocal • critical value

Objective

Build the skills to answer exam questions on **the modulus function, inequalities and symmetry** 、 .

You must be able to:

- solve equations and inequalities with $|x|$, by algebra and by graphs
- solve rational inequalities by combining into one fraction
- sketch $y = |f(x)|$, $y = f(|x|)$ and $y = \dfrac{1}{f(x)}$ from $y = f(x)$
- decide whether a function is odd, even or neither

1 Key knowledge and worked examples

■ Modulus, inequalities and symmetry

$|x| = x$ if $x \geq 0$, and $|x| = -x$ if $x < 0$. $|x - a| < b \Leftrightarrow a - b < x < a + b$.

graph	from $y = f(x)$
$y = \lvert f(x)\rvert$	reflect the parts **below** the x-axis upwards
$y = f(\lvert x\rvert)$	keep the right half; reflect it in the y-axis
$y = \dfrac{1}{f(x)}$	zeros of f become vertical asymptotes; a maximum of f becomes a minimum

Even: $f(-x) = f(x)$, symmetric in the y-axis. **Odd:** $f(-x) = -f(x)$, rotational symmetry about O.

Rational inequalities: never multiply by an expression that may be negative. Move everything to one side, make one fraction, and use a sign table.

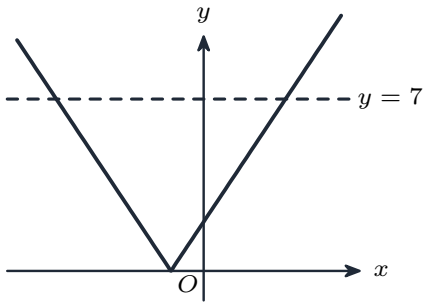
Worked example. $|2x - 1| = 5$: $2x - 1 = 5$ or $2x - 1 = -5$, so $x = \mathbf{3}$ or $x = \mathbf{-2}$.

exercises.lol exercises.com.cn

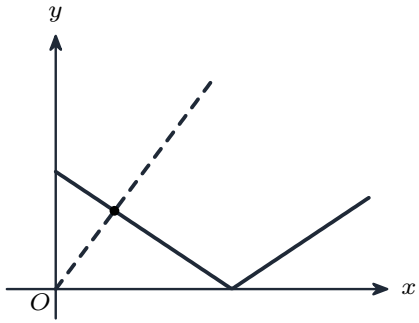
Answer shapes: split a modulus into cases, then check each answer in the original.

2 Practice

2.1 Solve $|3x + 2| = 7$. [2]

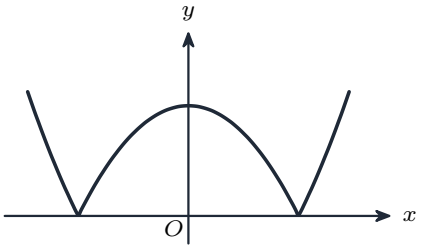


2.2 Solve $|x - 3| < 2x$. [3]



3 Exam-style questions

3.1 $f(x) = x^2 - 4$.



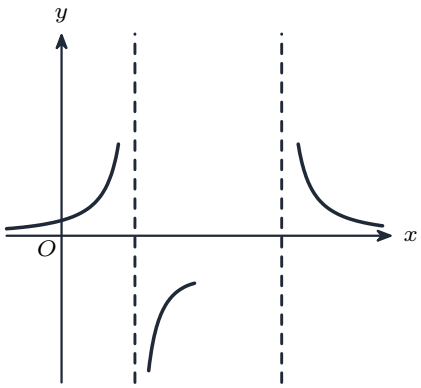
(a) Show that f is an even function. [2]

(b) Solve $|x^2 - 4| = 3$. [3]

(c) Write down the number of solutions of $|x^2 - 4| = 4$. [2]

3.2 (a) Solve $\frac{x+1}{x-2} \geq 2$. [4]

$h(x) = \frac{1}{x^2 - 4x + 3}$.



(b) Write down the equations of the vertical asymptotes of h. [2]

(c) Find the coordinates of the local maximum of h. [2]

exercises.lol exercises.com.cn

Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$3x + 2 = 7$, $x = \frac{5}{3}$; $3x + 2 = -7$, $x = \mathbf{-3}$

2.2

for $x < 3$: $3 - x < 2x$; critical value $x = 1$; $\mathbf{x > 1}$

3.1

(a) $f(-x) = (-x)^2 - 4 = x^2 - 4 = f(x)$, so f is even

(b) $x^2 - 4 = 3$ gives $x = \pm\sqrt{7}$; $x^2 - 4 = -3$ gives $x = \pm 1$

(c) $x = \pm\sqrt{8}$ and $x = 0$; **3** solutions

3.2

(a) $\dfrac{x+1}{x-2} - 2 \geq 0$; $\dfrac{5-x}{x-2} \geq 0$; critical values 2 and 5; $\mathbf{2 < x \leq 5}$

(b) $x = 1$; $x = 3$

(c) $x^2 - 4x + 3$ has its minimum -1 at $x = 2$; local maximum of h at $(\mathbf{2}, \mathbf{-1})$

exercises.lol exercises.com.cn