2.4 Polynomials: factor theorem and roots

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS polynomial • factor theorem • remainder theorem • root • repeated root • cubic

Objective

Build the skills to answer exam questions on **polynomials** : the factor and remainder theorems, factorising cubics, and the sum and product of roots.

You must be able to:

- use the factor and remainder theorems
- factorise a cubic and solve polynomial equations and inequalities
- use the sum and product of the roots of a polynomial
- sketch polynomial graphs, using repeated roots and end behaviour

1 Key knowledge and worked examples

Polynomials

result	statement
remainder theorem	$p(x) \div (x-a)$ leaves remainder $p(a)$
factor theorem	$(x-a)$ is a factor $\Leftrightarrow p(a)=0$
sum of roots of $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0$	$-\dfrac{a_{n-1}}{a_n}$
product of roots	$\dfrac{(-1)^n a_0}{a_n}$

Graphs: at a single root the graph crosses the axis; at a **double** root it **touches**; at a triple root it flattens as it crosses.

Worked example. $p(x) = x^3 - 2x^2 - 5x + 6$. $p(1) = 1 - 2 - 5 + 6 = 0$, so $(x-1)$ is a factor. Comparing coefficients:

$$p(x) = (x-1)(x^2 - x - 6) = \mathbf{(x-1)(x-3)(x+2)}$$

Check: sum of roots $1 + 3 - 2 = 2 = -\dfrac{-2}{1}$.

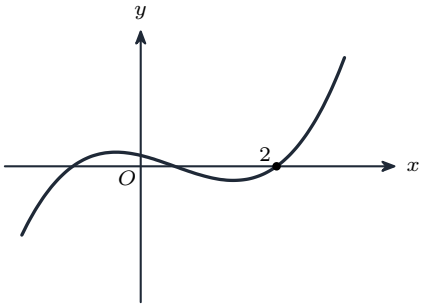
Answer shapes: show the value $p(a)$; for inequalities, use the roots and a sketch.

2 Practice

2.1 Find the remainder when $x^3 + 2x - 7$ is divided by $(x-2)$. [2]

$$x - 2 \overline{\left| \; x^3 + 0x^2 + 2x - 7 \right.}$$

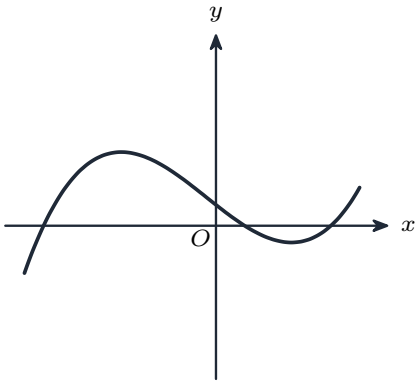
2.2 $p(x) = x^3 + ax^2 + bx - 6$. $(x-2)$ is a factor of $p(x)$, and the remainder when $p(x)$ is divided by $(x+1)$ is -12. Find a and b. [3]



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3 Exam-style questions

3.1 $p(x) = 2x^3 + x^2 - 13x + 6$.

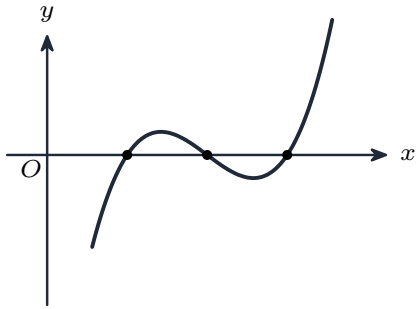


(a) Show that $(x - 2)$ is a factor of $p(x)$. [2]

(b) Factorise $p(x)$ fully. [3]

(c) Solve $p(x) < 0$. [2]

3.2 The equation $x^3 - 6x^2 + kx - 6 = 0$ has roots 1, 2 and γ.



(a) Use the sum of the roots to find γ. [2]

(b) Show that the product of the roots agrees with your answer to (a). [1]

(c) Find k. [2]

(d) Write down the y-intercept of $y = x^3 - 6x^2 + kx - 6$. [1]

(e) Solve $x^3 - 6x^2 + kx - 6 > 0$. [2]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$p(2) = 8 + 4 - 7 = \mathbf{5}$

2.2

$8 + 4a + 2b - 6 = 0$ and $-1 + a - b - 6 = -12$; $2a + b = -1$, $a - b = -5$; $a = \mathbf{-2}$, $b = \mathbf{3}$

3.1

(a) $p(2) = 16 + 4 - 26 + 6 = 0$, so $(x - 2)$ is a factor

(b) $p(x) = (x - 2)(2x^2 + 5x - 3)$; $= (\mathbf{x - 2})(\mathbf{2x - 1})(\mathbf{x + 3})$

(c) roots $-3,\ \frac{1}{2},\ 2$; $\mathbf{x < -3}$ or $\mathbf{\frac{1}{2} < x < 2}$

3.2

(a) $1 + 2 + \gamma = 6$; $\gamma = \mathbf{3}$

(b) product $= 1 \times 2 \times 3 = 6 = -\dfrac{-6}{1}$

(c) $k = 1 \cdot 2 + 1 \cdot 3 + 2 \cdot 3 = \mathbf{11}$

(d) $(0, -6)$

(e) $\mathbf{1 < x < 2}$ or $\mathbf{x > 3}$

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