2.2 Exponentials, logarithms and modelling

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS half-life

Objective

Exponentials, logarithms and modelling. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.

The method

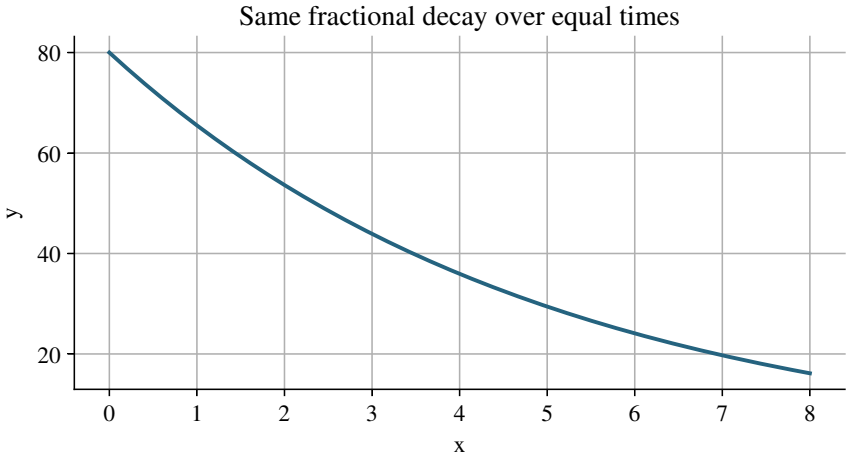
- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

A worked case

$$y = Ae^{kt}, \qquad \ln y = \ln A + kt$$

Example:

If y=80e^(-0.2t), y=40 gives e^(-0.2t)=1/2. Hence t=ln2/0.2≈3.466. For 3^x=20, x=ln20/ln3. Plotting ln y against t linearises this exponential model.



Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

Not true: For positive x and y, ln(x+y)=ln x+ln y. The log of a sum has no simple law; the product rule is ln(xy)=ln x+ln y.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

■ **Write from 2 to the power 8**

- Solve 2 to the power x = 256.
- The sum of ln 2 and ln 9 is ln 18, not ln of the sum.

2 Practice

2.1 Solve 2^x=32. Show the relation, substitution and units. [2]

2.2 Find ln(e^3). Show the relation, substitution and units. [2]

2.3 Find the half-life of 80e^(-0.2t), to three decimal places. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: For positive x and y, ln(x+y)=ln x+ln y. [2]

2.5 Why is ln(x+y)=ln x+ln y wrong? Choose one answer.

- **A** Logarithms apply only to integers, so ln(x+y) is undefined for most x and y
- **B** The law is ln(x+y)=ln x × ln y, so adding the logs gives the wrong value
- **C** The log of a SUM has no such law; the law covers products: ln(xy)=ln x+ln y [1]

3 Exam-style questions

3.1 A fictional quantity follows 80 exp(−0.2 t). Find when it reaches 20, retaining an exact expression; explain why the half-life does not depend on 80. [4]

Solutions

2.1 $32=2^5$, so x=5.

2.2 The natural logarithm undoes exponentiation: $\ln(e^3)=3$.

2.3 Set e^(-0.2t)=1/2. Then $t=\ln(2)/0.2\approx3.466$ to three decimal places.

2.4 False. The log of a sum has no simple law; the product rule is ln(xy)=ln x+ln y.

2.5 C Logarithms convert multiplication into addition: ln(xy)=ln x+ln y. Try x=y=1: ln $2\approx0.69$ but ln 1+ln 1=0, so the claimed law fails.

3.1 $t=\ln 4/0.2\approx6.931$ time units. Half-life ln 2/0.2 follows from a ratio 1/2; the initial multiplier cancels. Extrapolation assumes the exponential model continues.

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