1.5 Complex numbers and roots

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS complex conjugate

Objective

Complex numbers and roots. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.

The method

- Write $z=a+bi$ with $i^2=-1$.
- The conjugate is $a-bi$ and z times its conjugate$=a^2+b^2$.
- The modulus is $\sqrt{(a^2+b^2)}$; the argument needs the correct quadrant.
- Conjugate roots occur for polynomials with real coefficients.

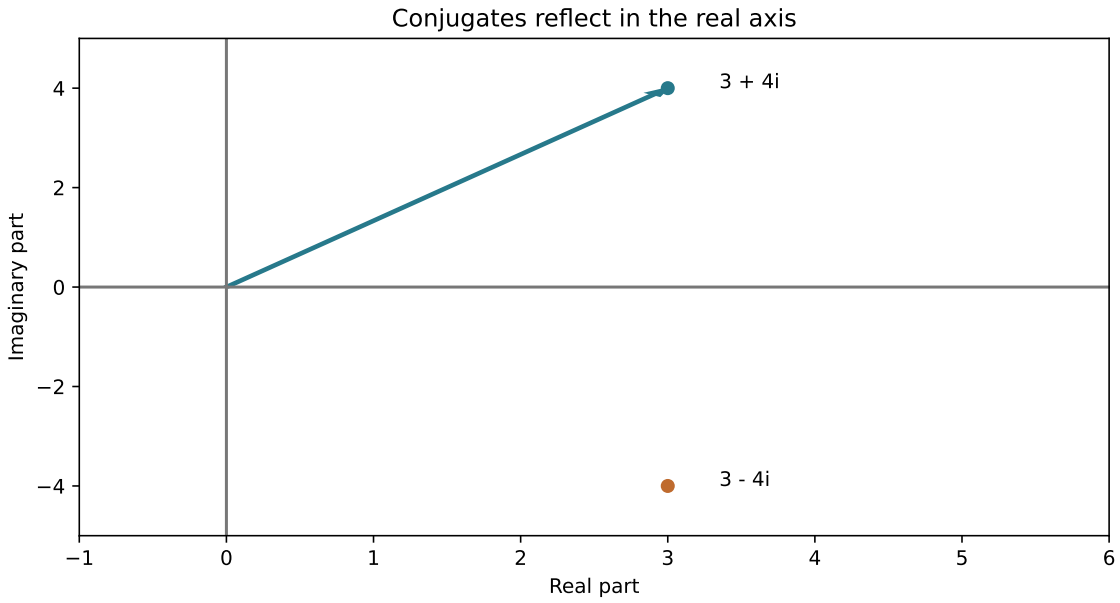
A worked case

$$z = a + bi, \qquad |z| = \sqrt{a^2 + b^2}, \qquad z\overline{z} = a^2 + b^2$$

Example:

For $z=3+4i$, $|z|=5$ and z conjugate$=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm2i$. Multiplying by the conjugate makes a complex denominator real.

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Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

Not true: For a+bi, its modulus is always a+b. The modulus is $\sqrt{(a^2+b^2)}$: for 3+4i it is 5, not 7.

Applying it

- Draw the real axis horizontally and imaginary axis vertically.
- For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Complex conjugate**: The number obtained by reversing the sign of the imaginary part.
- Write z=a+bi with i^2=-1.
- Watch for: the square root of a sum is not the sum of the square roots.

■ **Write from a modulus of 13**

- Find the modulus of 5 plus 12 i.
- The modulus is not the sum of the two parts.

2 Practice

2.1 Find |3+4i|. Show the relation, substitution and units. [2]

2.2 Find the real part of (3+4i)(3-4i). Show the relation, substitution and units. [2]

2.3 Find the real part of 1/(3+4i). Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: For a+bi, its modulus is always a+b. [2]

2.5 Why do we multiply 1/(3+4i) by (3-4i)/(3-4i)? Choose one answer.

- **A** It removes the real part, leaving a purely imaginary denominator
- **B** It doubles the modulus of the denominator, which makes the division easier
- **C** The conjugate product (3+4i)(3-4i)=25 is real, clearing the complex denominator [1]

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3 Exam-style questions

3.1 Find all complex roots of $z^2 = -8\,i$ in Cartesian form and verify them. [4]

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Solutions

2.1 Modulus=$\sqrt{(3^2+4^2)}=5$.

2.2 Conjugate multiplication gives $3^2+4^2=25$, with imaginary parts cancelling.

2.3 Multiply numerator and denominator by 3-4i: $1/(3+4i)=(3-4i)/25$. Its real part is $3/25=0.12$.

2.4 False. The modulus is $\sqrt{(a^2+b^2)}$: for 3+4i it is 5, not 7.

2.5 C z times its conjugate is $|z|^2=a^2+b^2$, a real number. Dividing by that real value leaves a clean a+bi form: (3-4i)/25.

3.1 Polar modulus 8 and argument$-\pi/2$ give roots modulus$\sqrt{8}$, arguments$-\pi/4$ and $3\pi/4$:2-2 i and $-2+2$ i. Squaring each gives -8 i. Both roots are needed.

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