1.4 Percentages, ratio and proportional reasoning

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS multiplier

Objective

Percentages, ratio and proportional reasoning. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier** : A factor that performs a percentage change in one multiplication.

The method

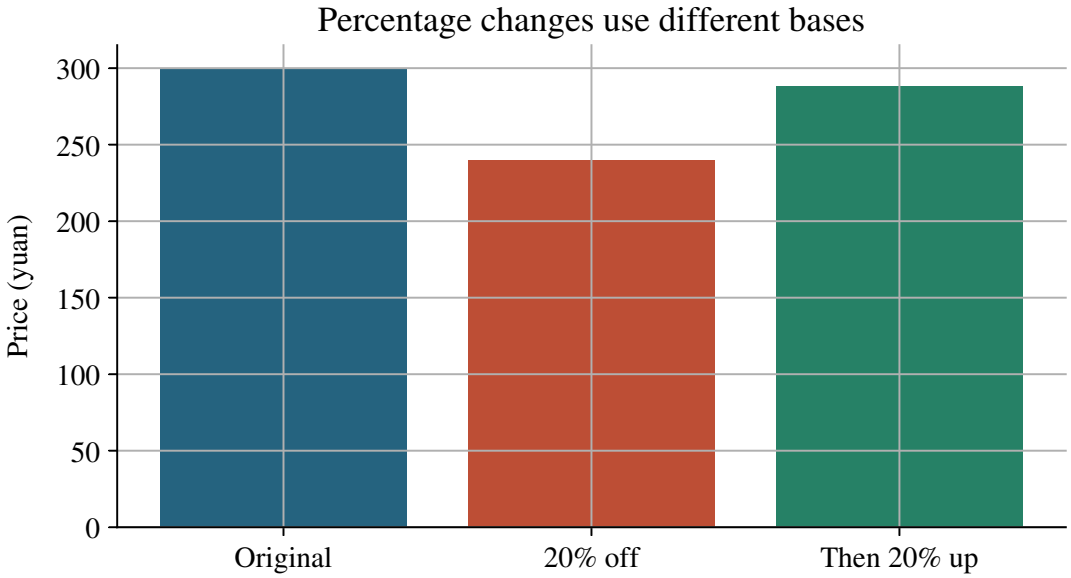
- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Reverse a percentage by dividing by the multiplier.
- In a ratio, first find the total number of parts.

A worked case

$$P_{\text{new}} = P_{\text{old}}\left(1 + \frac{r}{100}\right)$$

Example:

Let the original price be P. The model is sale price=0.8P. Hence P=240/0.8=300. A later 20% increase gives 240×1.2=288, so the two changes do not cancel.



Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100\times0.8\times1.2=96$, not 100.

Applying it

- For direct proportion use y=kx; for inverse proportion use y=k/x.
- Calculate k from a known pair before using a new value.
- State what you held constant.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Multiplier**: A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Watch for: A percentage uses a stated base.

■ **Write from a final price of 240**

- A price of 250 falls by 20 percent and then rises by 20 percent.
- Find the final price. The two changes do not cancel.

2 Practice

2.1 A price of 80 rises by 15%. Find the new price. Show the relation, substitution and units. [2]

2.2 A 25% discount leaves a price of 90. Find the original price. Show the relation, substitution and units. [2]

2.3 Share 84 in the ratio 2:5. Find the larger share. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Multiplier means a factor that performs a percentage change in one multiplication. [2]

2.5 A price falls 20% then rises 20%. Why is the final price not the starting price? Choose one answer.

- **A** The 20% rise is taken of the original price, so it cancels exactly
- **B** Percentage changes always cancel when they are equal and opposite
- **C** The second 20% acts on the reduced price, not the original [1]

3 Exam-style questions

3.1 A price rises 20% then falls 20%. Find the net change and the percentage increase needed to recover the original after that pair. [4]

Solutions

2.1 Multiply by 1.15: $80\times1.15=92$.

2.2 The sale price is 75% of the original: $90/0.75=120$.

2.3 There are seven ratio parts. The larger share is $84\times5/7=60$.

2.4 True. True: A factor that performs a percentage change in one multiplication.

2.5 C A 20% decrease multiplies by 0.8 and the later 20% increase multiplies that result by 1.2: $0.8\times1.2=0.96$. Each percentage uses a different base.

3.1 Factor $1.2\times0.8=0.96$, a 4% fall. Recovery factor $1/0.96$, giving 4.1667% increase. Both percentage changes use different bases.