1.2 Indices, surds and standard form

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS index

Objective

Indices, surds and standard form. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

The method

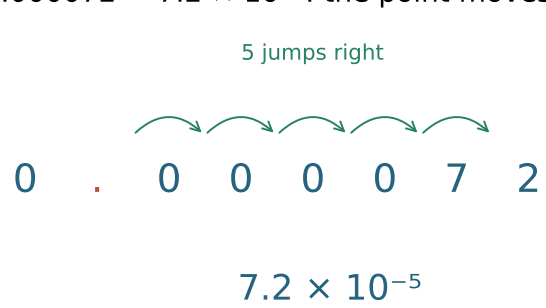
- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has 1≤a<10.

A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

Example:

0.000072=7.2×10^(-5). Also 16^(3/4)=(16^(1/4))^3=2^3=8. Simplify √72=6√2, then rationalise 1/√2=√2/2.



Where it goes wrong

- Index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.
- Do not round a surd when an exact answer is requested.

Not true: For every positive a, a^2+a^3 equals a^5. Index laws only add powers when the terms are multiplied: $2^2+2^3=12$, but $2^5=32$.

Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Index**: The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.

■ **Write from 8 to the power 2/3**

- Evaluate 8 to the power 2/3.
- A squared plus a cubed is not a to the fifth.

2 Practice

2.1 Evaluate 16^(3/4). Show the relation, substitution and units. [2]

2.2 Evaluate 2^(-3). Show the relation, substitution and units. [2]

2.3 Write 7.2×10^(-5) as a decimal. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: For every positive a, a^2+a^3 equals a^5. [2]

2.5 Why is 16^(3/4) equal to 8? Choose one answer.

- **A** The numerator cube applies after the fourth root: $(16^{(1/4)})^3=2^3$
- **B** Multiply the base by the index first: 16×3/4=12, then round to 8
- **C** The fourth root applies after cubing: $16^3=4096$, and $\sqrt[4]{4096}$ is 8 [1]

3 Exam-style questions

3.1 Solve x^(3/2)=27 over the real domain used for this power. Check the result and explain why a negative candidate is invalid. [4]

Solutions

2.1 $16=2^4$, so $16\hat{}(3/4)=2^3=8$.

2.2 A negative index gives a reciprocal: $2\hat{}(-3)=1/8=0.125$.

2.3 Move the decimal point five places left: $7.2\times10\hat{}(-5)=0.000072$.

2.4 False. Index laws only add powers when the terms are multiplied: $2^2+2^3=12$, but $2^5=32$.

2.5 A A fractional index means root then power: $16\hat{}(3/4)=(\sqrt[4]{16})^3=2^3=8$. Multiplying 16 by 3/4 confuses the index with a factor.

3.1 Take the nonnegative real square-root convention: $x\geq0$. $x=27\hat{}(2/3)=9$; $9\hat{}(3/2)=27$. A negative x has no real square root under this convention.