1.1 Exact arithmetic and estimation

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS prime factor

Objective

Exact arithmetic and estimation. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

A worked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \mathrm{HCF}(a,b) = \prod p_i^{\min(\alpha_i,\beta_i)}$$

Example:

72=2^3×3^2 and 90=2×3^2×5. Their HCF is 2×9=18. Make 18 bags with 4 pencils and 5 pens each. Their LCM is 2^3×3^2×5=360.

$72 = 2^3 \times 3^2$

$90 = 2 \times 3^2 \times 5$

Shared smallest powers: $2 \times 3^2 = 18$

Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

Not true: The HCF of two positive integers is always larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.

Key:

- **Prime factor**: A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

■ **Write from an HCF of 6**

- Find the HCF and LCM of 18 and 30.
- The HCF is smaller than both integers.

2 Practice

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.3 Work out 3/4 + 5/6 as a decimal. Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.5 Which statement correctly relates HCF and LCM of two positive integers? Choose one answer.

- **A** The LCM always equals their product
- **B** The HCF is always the larger of the two numbers
- **C** The HCF divides both; the LCM is a multiple of both [1]

3 Exam-style questions

3.1 A supplier has 84 pencils and 126 pens. Find the greatest number of identical bags without leftovers and the contents of each; then explain why the LCM answers a different question. [4]

Solutions

2.1 $48=2^4\times3$ and $60=2^2\times3\times5$. The shared smallest powers give HCF$=2^2\times3=12$.

2.2 $8=2^3$ and $12=2^2\times3$. The largest prime powers give LCM$=2^3\times3=24$.

2.3 Use denominator 12: $3/4+5/6=9/12+10/12=19/12$.

2.4 False. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

2.5 C The highest common factor must divide each number, and the lowest common multiple must be a multiple of each. Only when the numbers share no factor does the LCM equal the product.

3.1 HCF=42, each bag 2 pencils and 3 pens. LCM=252 is a common multiple, useful for matching repeated quantities, not the greatest number of bags dividing both stocks.