

Exercise walk-through

5.2

Integrals, area and accumulation

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

5.2 Integrals, area and accumulation

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS antiderivative

Objective

Integrals, area and accumulation. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** : A function whose derivative equals the given integrand.

■ The method

- For $n \neq -1$, the integral of ax^n is $ax^{(n+1)}/(n+1)+C$.
 - A definite integral is signed accumulation.
 - Split at sign changes when total area or distance is required.
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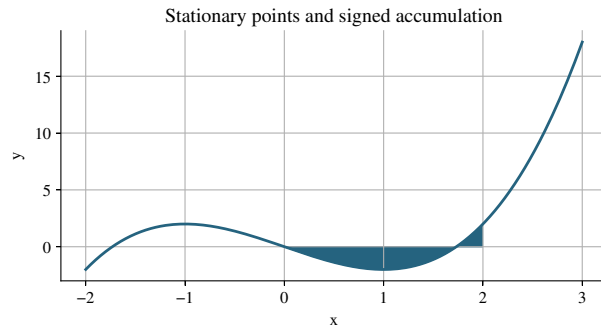
■ A worked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

Example:

For $v(t)=3t^2-3$ over $0 \leq t \leq 2$, displacement= $[t^3-3t]_0^2=2$. Since v changes sign at $t=1$, distance= $-[t^3-3t]_0^1+[t^3-3t]_1^2=2+4=6$. The constants cancel only for a definite

integral.



■ **Where it goes wrong**

- The integral of $1/x$ is $\ln|x|+C$, not the power rule with $n=-1$.
- Signed area can be zero even when the total enclosed area is positive.

Not true: A definite integral always equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

■ **Applying it**

- Identify what the integral means and include the correct units.
 - A rate measured per second integrates to the underlying quantity, not to another rate.
 - Differentiate an antiderivative to check it.
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■ **In your course**

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Antiderivative:** A function whose derivative equals the given integrand.
 - For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1)+C$.
 - Watch for: the integral of $1/x$ is $\ln|x|+C$, not the power rule with $n=-1$.
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■ Write from an integral of 9

- Find the integral of $2x$ from 0 to 3.
- For $v = t$ minus 2, from $t = 0$ to 4, find the displacement and the distance. A signed integral is not the total distance.

Exercises 2.1-2.3

2.1 Find the integral of $3x^2$ from 0 to 2. Show the relation, substitution and units. [2]

2.2 For $v=3t^2-3$, find displacement from $t=0$ to 2. Show the relation, substitution and units. [2]

2.3 For that motion, find total distance from $t=0$ to 2. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find the integral of $3x^2$ from 0 to 2. Show the relation, substitution and units. [2]

2.1 An antiderivative is x^3 . Subtract its values: $2^3 - 0^3 = 8$.

2.2 For $v = 3t^2 - 3$, find displacement from $t = 0$ to 2. Show the relation, substitution and units. [2]

2.2 Displacement = $[t^3 - 3t]$ from 0 to 2 = $8 - 6 = 2$.

2.3 For that motion, find total distance from $t = 0$ to 2. Show the relation, substitution and units. [2]

2.3 Velocity changes sign at $t = 1$. The two positive distances are 2 and 4, giving 6.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Antiderivative means a function whose derivative equals the given integrand. [2]

2.5 For $v(t)=3t^2-3$ on $0 \leq t \leq 2$, why are displacement and total distance different? Choose one answer.

- **A** Distance equals displacement whenever the velocity is a polynomial
- **B** Displacement ignores time, so it cannot be found by integrating $v(t)$
- **C** The velocity changes sign at $t=1$, so the signed integral cancels part of the travel [1]

3.1 For velocity $v=t-2$ on $0 \leq t \leq 5$, find displacement and total distance and explain the distinction. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Antiderivative means a function whose derivative equals the given integrand. [2]

2.4 True. True: A function whose derivative equals the given integrand.

2.5 For $v(t)=3t^2-3$ on $0 \leq t \leq 2$, why are displacement and total distance different? Choose one answer.

- **A** Distance equals displacement whenever the velocity is a polynomial
- **B** Displacement ignores time, so it cannot be found by integrating $v(t)$
- **C** The velocity changes sign at $t=1$, so the signed integral cancels part of the travel [1]

2.5 C The signed integral over the whole interval gives displacement 2. The motion reverses at $t=1$, so total distance needs the split: 2 backwards then 4 forwards, totalling 6.

3.1 For velocity $v=t-2$ on $0 \leq t \leq 5$, find displacement and total distance and explain the distinction. [4]

3.1 Integral $[t^2/2 - 2t]_0^5 = 2.5$. Sign changes at 2; signed integrals are -2 then 4.5 , so distance 6.5 . Units follow velocity/time units. Distance integrates $|v|$.