

Exercise walk-through

4.3

Binomial, normal and Poisson models

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

4.3 Binomial, normal and Poisson models

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS expected value

Objective

Binomial, normal and Poisson models. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** : The probability-weighted mean of a

random variable.

■ The method

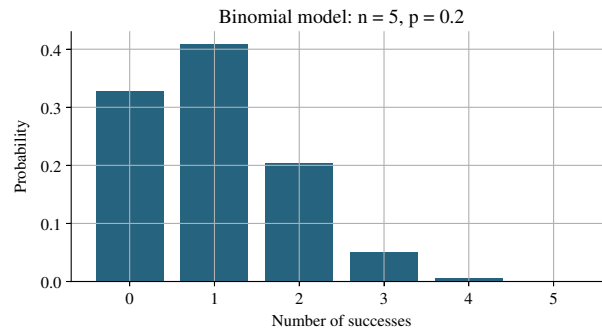
- A binomial model needs fixed n , independent trials, two outcomes and constant p .
 - $E(X)=np$ and $\text{Var}(X)=np(1-p)$.
 - For a normal model use $z=(x-\mu)/\sigma$.
 - A Poisson model describes counts with a constant rate and appropriate independence assumptions.
-

■ A worked case

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad X \sim B(n, p)$$

Example:

For X binomial(5,0.2), $P(X=0)=0.8^5=0.32768$, $E(X)=1$ and $\text{Var}(X)=0.8$. For a normal quantity with $\mu=100, \sigma=15$, the value 130 has $z=2$. A Poisson mean of 3 per hour gives mean 6 over two hours.



■ Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

Not true: Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

■ Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X < k)$, $P(X \leq k)$ and a tail complement.
- State assumptions in context.

■ In your course

- Analysis and Approaches HL, the course first assessed in 2021.

Key:

- **Expected value:** The probability-weighted mean of a random variable.
 - A binomial model needs fixed n , independent trials, two outcomes and constant p .
 - Watch for: not every count is binomial: changing p or dependence can invalidate it.
-

■ **Write from a z-score of 3**

- A value 82 comes from a distribution with mean 70 and standard deviation 4.
- Find the z-score. A binomial count needs independent trials.

Exercises 2.1-2.3

2.1 For X binomial(5,0.2), find $P(X=0)$. Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.2 Find $\text{Var}(X)$ for X binomial(5,0.2). Show the relation, substitution and units. [2]

2.3 For $\mu=100$ and $\sigma=15$, find the z-score of 130. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For X binomial(5,0.2), find $P(X=0)$. Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.1 No successes means five failures: $0.8^5=0.32768$.

2.2 Find $\text{Var}(X)$ for X binomial(5,0.2). Show the relation, substitution and units. [2]

2.2 Binomial variance= $np(1-p)=5 \times 0.2 \times 0.8=0.8$.

2.3 For $\mu=100$ and $\sigma=15$, find the z-score of 130. Show the relation, substitution and units. [2]

2.3 Standardise: $z=(130-100)/15=2$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every count of successes has a binomial distribution regardless of dependence. [2]

2.5 A count of successes fails the binomial model when which condition breaks? Choose one answer.

- **A** The number of trials is fixed in advance, so the count cannot be binomial
- **B** The success probability changes between trials or trials are dependent
- **C** Each trial has exactly two outcomes, which rules out the binomial model [1]

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every count of successes has a binomial distribution regardless of dependence. [2]

2.4 False. The binomial model needs independent trials with a fixed probability; dependent trials break it.

2.5 A count of successes fails the binomial model when which condition breaks? Choose one answer.

- **A** The number of trials is fixed in advance, so the count cannot be binomial
- **B** The success probability changes between trials or trials are dependent
- **C** Each trial has exactly two outcomes, which rules out the binomial model [1]

2.5 B The binomial model needs fixed n , independent trials, two outcomes and a constant p . Sampling without replacement makes trials dependent, so the binomial formula no longer applies exactly.

Exercise 3.1

3.1 For $X \sim \text{Binomial}(5, 0.2)$, find $P(X \geq 1)$ and the mean. Explain one situation in which the binomial model would be inappropriate. [4]

Exercise 3.1

3.1 For $X \sim \text{Binomial}(5, 0.2)$, find $P(X \geq 1)$ and the mean. Explain one situation in which the binomial model would be inappropriate. [4]

3.1 $1 - 0.8^5 = 0.67232$; mean $np = 1$. Trials must be independent with constant success probability and fixed count; depletion or correlated outcomes can violate this.