Exercise walk-through

# 4.2 Probability, trees and conditional reasoning

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

INTERNATIONAL BACCALAUREATE · IB DIPLOMA ·
MATHEMATICS: ANALYSIS AND APPROACHES · HL • EXERCISE SHEET

# 4.2 Probability, trees and conditional reasoning

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Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** conditional probability

## Objective

Probability, trees and conditional reasoning. Original classroom practice for IB HL; not an official IB paper.

## 1 Key knowledge and worked examples

### ■ What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event

after restricting the sample space to a stated condition.

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## The method

- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

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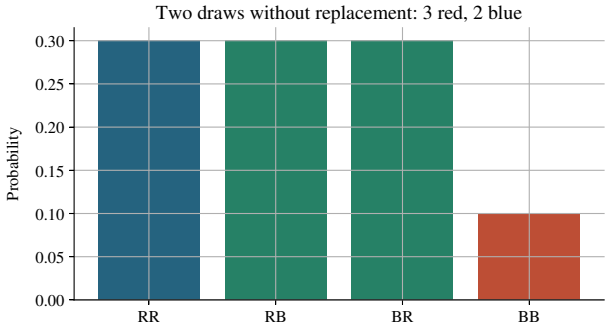
## A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

**Example:**

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/
If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.

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### Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

**Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

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### Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

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### In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and $\pi$ exact and show each step. On calculator papers, still write the expression before its value.

**Key:**

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

- **Write from a conditional probability of 0.64**
- P(A and B) is 0.16 and P(B) is 0.25.
- Find P(A given B). Mutually exclusive events that can happen are not independent.

# Exercises 2.1·2.3

**2.1** Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

**2.2** Find P(A given B) if $P(A \cap B)=0.12$ and $P(B)=0.3$. Show the relation, substitution and units. [2]

**2.3** Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

# Exercises 2.1·2.3

**2.1** Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

**2.1** Without replacement, $P(RR)=3/5\times2/4=0.3$.

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**2.2** Find P(A given B) if $P(A\cap B)=0.12$ and $P(B)=0.3$. Show the relation, substitution and units. [2]

**2.2** $P(A|B)=P(A\cap B)/P(B)=0.12/0.3=0.4$.

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**2.3** Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

**2.3** Add the two orders: $3/5\times2/4+2/5\times3/4=0.6$.

# Exercises 2.4·2.5

**2.4** Decide whether this claim is true and justify: Conditional probability means the probability of an event after restricting the sample space to a stated condition. [2]

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**2.5** Two disjoint events A and B both have positive probability. Can they be independent? Choose one answer.

- **A** Yes: independence is automatic for all events
- **B** Yes: disjoint and independent mean the same thing
- **C** No: if A happens, P(B) drops to 0, so knowing A changes B [1]

# Exercises 2.4·2.5

**2.4** Decide whether this claim is true and justify: Conditional probability means the probability of an event after restricting the sample space to a stated condition. [2]

**2.4** True. True: The probability of an event after restricting the sample space to a stated condition.

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**2.5** Two disjoint events A and B both have positive probability. Can they be independent? Choose one answer.

- **A** Yes: independence is automatic for all events
- **B** Yes: disjoint and independent mean the same thing
- **C** No: if A happens, P(B) drops to 0, so knowing A changes B [1]

**2.5 C** Disjoint means A and B cannot happen together, so P(B given A)=0≠P(B). Independence requires that knowing one does not change the other - disjoint events with positive probability always fail that.

# Exercise 3.1

**3.1** A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

# Exercise 3.1

**3.1** A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

**3.1** P(RR)=3/5×2/4=0.3. P(at least oneR)=1−2/5×1/4=0.9. Conditional probability=0.3/0.9=1/3. The conditioning changes the sample space.