

Exercise walk-through

3.3

Radians, identities and trigonometric equations

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

3.3 Radians, identities and trigonometric equations

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS radian

Objective

Radians, identities and trigonometric equations. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** : The angle subtended by an arc whose length equals

the radius.

■ The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
 - Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
 - List every solution in the stated interval.
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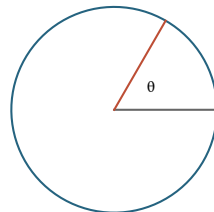
■ A worked case

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2 \theta + \cos^2 \theta = 1$$

Example:

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

■ **Where it goes wrong**

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.

Not true: The principal inverse sine value is always the only solution in a full turn. Sine takes most values twice in a full turn: $\sin x=0.5$ gives 30° and 150° .

■ **Applying it**

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

■ **In your course**

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Radian:** The angle subtended by an arc whose length equals the radius.
 - For θ in radians, arc length $s=r\theta$ and sector area $A=\frac{1}{2}r^2\theta$.
 - Watch for: A calculator's inverse sine gives one principal value.
-

■ Write from an arc length of 3

- A circle has radius 6 and the angle is 0.5 radians.
- Find the arc length and the sector area. The principal inverse sine is not the only solution in a full turn.

Exercises 2.1-2.3

2.1 For $r=4$ and $\theta=0.5$ radians, find arc length. Show the relation, substitution and units. [2]

2.2 For $r=4$ and $\theta=0.5$ radians, find sector area. Show the relation, substitution and units. [2]

2.3 How many solutions does $\sin x=0.5$ have on $0 \leq x < 2\pi$? Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $r=4$ and $\theta=0.5$ radians, find arc length. Show the relation, substitution and units. [2]

2.1 Arc length $= r\theta = 4 \times 0.5 = 2$.

2.2 For $r=4$ and $\theta=0.5$ radians, find sector area. Show the relation, substitution and units. [2]

2.2 Sector area $= r^2\theta/2 = 16 \times 0.5/2 = 4$.

2.3 How many solutions does $\sin x = 0.5$ have on $0 \leq x < 2\pi$? Show the relation, substitution and units. [2]

2.3 The two solutions are $\pi/6$ and $5\pi/6$ in the stated interval.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: The principal inverse sine value is always the only solution in a full turn. [2]

2.5 Solving $\sin x = 1/2$ on $0 \leq x < 2\pi$, why are there two solutions? Choose one answer.

- **A** Every equation has exactly two solutions
- **B** Sine is positive in two quadrants, so $\pi/6$ and $5\pi/6$ both work
- **C** The calculator inverse gives both values at once [1]

3.1 Solve $2 \sin^2 x - \sin x - 1 = 0$ for $0 \leq x < 2\pi$, showing why no solutions are lost. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: The principal inverse sine value is always the only solution in a full turn. [2]

2.4 False. Sine takes most values twice in a full turn: $\sin x = 0.5$ gives 30° and 150° .

2.5 Solving $\sin x = 1/2$ on $0 \leq x < 2\pi$, why are there two solutions? Choose one answer.

- **A** Every equation has exactly two solutions
- **B** Sine is positive in two quadrants, so $\pi/6$ and $5\pi/6$ both work
- **C** The calculator inverse gives both values at once [1]

2.5 B The inverse sine returns only the principal value $\pi/6$. Sine is also positive in the second quadrant, giving $\pi - \pi/6 = 5\pi/6$. Both must be listed within the stated interval.

3.1 Solve $2 \sin^2 x - \sin x - 1 = 0$ for $0 \leq x < 2\pi$, showing why no solutions are lost. [4]

3.1 Factor $(2 \sin x + 1)(\sin x - 1) = 0$. $\sin x = 1$ gives $\pi/2$; $\sin x = -1/2$ gives $7\pi/6, 11\pi/6$. Both allowed roots are considered and the endpoint interval is respected.